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How to prove it a structured approach 3rd edition solutions

Structured approach example. How to prove it a structured approach 3rd edition. How to prove it a structured approach 3rd edition solutions pdf. Structured test example. Structured approach meaning. How to prove it a structured approach.

Numerade, a math education platform, aims to improve educational levels globally by creating video tutorials that can be used for years to come. Dr. Mei Lin Chen, a renowned mathematician, emphasizes the importance of proof techniques in advanced mathematics and theoretical computer science. To help students overcome obstacles, this best-selling textbook's third edition teaches readers how to read and write proofs, featuring over 150 new exercises and a chapter on number theory. Perfect for self-study or as a supplement for courses like discrete math or foundations of computing, this text guides students through the world of advanced mathematics using various techniques. It covers essential topics such as logic, set theory, relations, functions, cardinality, and more. The book is designed to be systematic and thorough, allowing readers to construct complex proofs by combining different techniques. Daniel J. Velleman, a prominent mathematician, contributed to this text. He has written several books on mathematics, including "Which Way Did the Bicycle Go?" and "Philosophies of Mathematics". The statement P(x) true can be made by letting x stand for an arbitrary object and proving P(x). If x is already being used in the proof for something, then choose an unused variable, say y, for the arbitrary object, and prove P(y). To prove a goal of the form ∃x P(x), let x stand for some object that satisfies P(x) and then prove P(x). If there are no free variables in the statement, it is either true or false. A set of all possible values of variables is called the universe of discourse. Or variables range over this universe. In general, y ∈ {x ∈ A | P(x)} means the same thing as y ∈ A ∧ P(y). The solutions to the given problems are: (a) D(6, 3) ∧ D(9, 3) ∧ D(15, 3) (b) D(x, 2) ∧ D(x, 3) ∧ ¬D(x, 4) (c) (¬P(x) ∧ P(y)) ∨ (P(x) ∧ ¬P(y)) (a) M(x) ∧ M(y) ∧ (T(x, y) ∨ T(y, x)) (b) [(B(x) ∨ B(y)) ∧ (R(x) ∨ R(y)] (c) [(B(x) ∧ R(x)) ∨ (B(y) ∧ R(y)] (a) {x | x is a planet} (b) {x | x is a university} (c) {x | x is a state in US} (d) {x | x is a province in Canada} (a) {x^2 | x > 0 and x ∈ ℕ} (b) {2^x | x ∈ ℕ} (c) {x ∈ ℕ | 10 ≤ x ≤ 19} (a) -3 ∈ {x ∈ ℝ | 13 - 2x > 1} ⇒ -3 ∈ ℝ ∧ 19 > 1 (b) 4 ∈ {x ∈ ℝ+ | 13 - 2x > 1} ⇒ 4 ∈ ℝ+ ∧ 5 > 1 (c) 5 ∉ {x ∈ ℝ | 13 - 2x > c} ⇒ ¬{5 ∈ ℝ ∧ 3 > c} (a) (w ∈ ℝ) ∧ (13 - 2w > c) (b) (4 ∈ ℝ) ∧ (13 - 2 × 4 ∈ P) ⇒ (4 ∈ ℝ) ∧ (5 ∈ P) (c) (4 ∈ P) ∧ (13 - 2 × 4 > 1) ⇒ (4 ∈ P) ∧ (5 > 1) (a) {Conrad Hilton Jr., Michael Wilding, Michael Todd, Eddie Fisher, Richard Burton, John Warner, Larry Fortensky} (b) {∨, ∧, ¬} (c) {Daniel Velleman} (a) {1, 3} (b) φ (c) {x ∈ ℝ | x^2 < 25} To prove a goal of the form ∀x P(x): Let x stand for an arbitrary object and prove P(x). If x is already being used in the proof for something, then choose an unused variable, say y, for the arbitrary object, and prove P(y). If there are no free variables in the statement, it is either true or false. Let x be arbitrary. [Proof of $P(x)$ goes here.] Since x was arbitrary, we can conclude that $\forall x P(x)$. To prove a goal of the form $\exists x P(x)$: Try to find a value of x for which P(x) will be true. Then start the proof with "Let x = (the value decided)" and proceed to prove P(x) for this value of x. Scratch work will look as: Given: Goal: $\exists x P(x)$ It will be transformed to: Given: Goal: $P(x)$ x = (the value decided) Formal proof will look as: Let x = (the value decided). [Proof of $P(x)$ goes here.] Thus $\exists x P(x)$. To use a given of the form $\exists x P(x)$: Introduce a new variable, say x0, into the proof for which P(x0) is true. Thus it can be assumed that P(x0) is true. This rule of inference is called existential instantiation. Using a given of the form $\exists x P(x)$ is very different from proving a goal of the form $\exists x P(x)$, because when using a given of the form $\exists x P(x)$, there is no choosing for the value of x. To use a given of the form $\forall x P(x)$: Plug in any value, say a, for x and use this given to conclude that P(a) is true. This rule is called universal instantiation. It will be useful for cases when particular value, say a, of x is already known or considered and thus it can be assumed that P(a) is true. Solution 1: Suppose $\exists x (P(x) → Q(x))$ is true at x = x0. Thus we have P(x0) → Q(x0). Suppose ∀x P(x). Then P(x0) is true. Since P(x0) → Q(x0) and P(x0), then Q(x0) is true. Thus $\exists x Q(x)$ is true. It follows that ∀x P(x) → ∃Q(x) is true. Solution 2: Suppose x is arbitrary and x ∈ A ∩ B, then x ∈ A and x ∈ B. Suppose x ∉ C. Since x ∈ B and x ∉ C, it follows that x ∈ B \ C. Now since x ∈ A and x ∈ B \ C, it follows that x ∈ (A ∩ (B \ C)). But A ∩ (B \ C) = φ. Thus the assumption x ∉ C is not correct. It follows that if x ∈ A ∩ B, then x ∈ C. As x is arbitrary, we have A ∩ B ⊆ C. Solution 3: Suppose x ∈ A and x ∈ C. This contradicts the assumptions that x ∈ A ∩ C. As x is arbitrary, A ∩ C = φ. Solution 4: Suppose x ∈ P(A). Then x ⊆ A. Since it is given that A ⊆ P(A), it follows that x ⊆ P(A). Thus if x ∈ P(A) then x ∈ P(P(A)). As x is arbitrary, we get P(A) ⊆ P(P(A)). Solution 5: (a) Empty set: φ. (b) No, there is no other such set. Solution 6: (a) Suppose y = (2x + 1) / (x - 1). Putting the value of y in (y + 1) / (y - 2). We get: = (2x + 1 + x - 1) / (2x + 1 - 2x + 2) = (3x) / (3) Thus ∃y ((...) = x. Given equation $\frac{y+1}{y-2}=x$, solving for y yields $y=\frac{2x+1}{x-1}$, which implies that $x \neq 1$. Then, it is solved that if $x>2$, then $y+\frac{1}{y}=x$. Additionally, some set theory solutions are given: $\cup\mathcal{F} \subseteq A$ and $\cap\mathcal{F}\subseteq B$; also, $\phi=\cap\mathcal{F}$, and $\cup\mathcal{F} \subseteq \cup\mathcal{G}$ and $\cap\mathcal{G}\subseteq\cap\mathcal{F}$. Lastly, it is shown that ${\cup}_{i \in I} \mathcal{P}(A_i) \subseteq \mathcal{P}({\cup}_{i \in I} A_i)$ for any sets $A_1,A_2,...$ **Mathematical Set Theory** The text discusses various set theory problems and solutions. * **Intersection of Power Sets**: If X is the intersection of power sets $\cap_{i \in I} \mathcal P(A_i)$, then for any $x \in X$, x is a subset of all A_i. Therefore, $x = \cap_{i \in I} A_i$. This implies that $\cap_{i \in I} A_i \in \cap_{i \in I} \mathcal P(A_i)$. * **Union and Intersection**: If $\mathcal F \subseteq \mathcal P(B)$, then all sets in $\mathcal F$ are subsets of B. Therefore, if $x \in \cup \mathcal F$, then $x \in B$, implying that $\cup \mathcal F \subseteq B$. Similarly, if all sets in $\mathcal F$ are subsets of every set in $\mathcal G$, then if $x \in \cup \mathcal F$, then x is an element of all sets in $\mathcal G$, implying that $\cup \mathcal F \subseteq \cap \mathcal G$. * **Divisibility**: Given that $b = ma$ and $c = na$, where m, n are integers, it can be shown that $a \vert (b+c)$. * **Proof Analysis**: The text also provides analysis of various proof mistakes, including: + Assuming $x \in B$ when starting with $x \in A$. + Fixing the value of y instead of considering an arbitrary value for x. + Failing to consider boundary cases (e.g., the empty set) in a theorem. Overall, the text covers various topics in mathematical set theory, including intersection and union operations, divisibility, and proof analysis. The equation F contains three sets, phi, {1,2}, and {3,5}. Similarly, G has three sets, phi, {4}, and {8,9}. The intersection of F and G is just phi, so they are not disjoint in the classical sense. However, when taking the union of F and G and then finding their intersection, we get an empty set, which means they are indeed disjoint. A counterexample to this proof involves considering x and y as distinct elements rather than equal ones. By setting x = 1 and y = 0, it's clear that the theorem isn't universally true. To further illustrate the concept of disjoint sets, let's consider three real numbers, x, y, and z. If we take the expression (x + z)^2 - (x^2 + z^2), it simplifies to 2xz. Now, if we set y = 2x, then yz becomes 2xz as well. This demonstrates that yz is indeed equal to (x + z)^2 - (x^2 + z^2). There's an interesting parallel between trying to prove a statement for all x (denoted ∀x P(x)) and proving it exists for some x (denoted ∃x P(x)). In the first case, we introduce a generic variable, say x, and proceed with our proof. Conversely, when given that something holds true for some value of x (∃x P(x)), we introduce a specific instance where this is true (let's call it x0) and use this to drive forward our argument. The approach is similar in both scenarios; we're merely starting from different perspectives. Similarly, if the goal is to prove that there exists some x for which P(x) holds (∃x P(x)), then we seek out a specific instance where this is true. On the other hand, when given that something is true for all x (∀x P(x)), our strategy involves finding an example value of x that can help us further with the proof.