Dividing Using Reciprocals

In the previous lesson, we learned that a fraction is the same as a whole number divided by another. Now let's see how to divide a fraction by a fraction. When dividing a fraction by a fraction, we have to use what is called the **RECIPROCAL**.

Reciprocal

To find the reciprocal of a fraction, switch the places of the numerator and denominator.

$$\frac{3}{4} \xrightarrow{\text{reciprocal}} \frac{4}{3}$$

Why does this work? Remember that division is the opposite of multiplication, and the opposite of a fraction is called its reciprocal. By changing two parts of the equation to their opposites, we are actually solving the same equation!

Example 1: Find the reciprocal of each fraction:

a) $\frac{3}{7}$

To find the reciprocal of a fraction, just switch the numerator and the denominator's places. 3 becomes the denominator and 7 becomes the numerator.

$$\frac{3}{7} \rightarrow \frac{7}{3}$$

b) $\frac{1}{8}$

Once again, switch the numerator's place with the denominator.

$$\frac{1}{8} \rightarrow \frac{8}{1}$$

We should always simplify our fractions whenever possible. $\frac{8}{1}$ is actually the whole number 8, so the reciprocal is 8.

c) 9

Like the question above, 9 is actually equal to the fraction $\frac{9}{1}$. We can then take the reciprocal of this fraction.

$$\frac{9}{1} \rightarrow \frac{1}{9}$$

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To divide fractions, change the **divisor** to its **reciprocal** and then multiply instead.

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3}$$

Example 2: Divide the following fractions:

a)
$$\frac{1}{3} \div \frac{3}{4}$$

We need to change the division sign to multiplication and the second fraction to its reciprocal. The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$, so:

$$\frac{1}{3} \div \frac{3}{4} = \frac{1}{3} \times \frac{4}{3}$$

Now multiply the fraction. First we multiply the numerators: 1 times 4 is 4. Then we multiply the denominators: 3 times 3 is 9.

$$\frac{1}{3} \times \frac{4}{3} = \frac{4}{9}$$

b)

$$\frac{3}{8} \div \frac{4}{6}$$

The reciprocal of $\frac{4}{6}$ is $\frac{6}{4}$, so $\quad \frac{3}{8} \div \frac{4}{6} = \frac{3}{8} \times \frac{6}{4}$

Then we multiply $\frac{3}{8} \times \frac{6}{4} = \frac{18}{32}$

Don't forget to reduce the answer if possible! Both the numerator and denominator have a common factor of 2, so we can simplify:

$$\frac{18 \div 2}{32 \div 2} = \frac{9}{16}$$

Example 3:

Leah is painting birdhouses for her backyard. She has $\frac{3}{6}$ of a litre of paint left. Each birdhouse requires $\frac{1}{16}$ of a litre of paint. How many birdhouses can she paint?

Number of birdhouses = all the paint ÷ paint per birdhouse

$$= \frac{3}{6} \div \frac{1}{16}$$

First, we take the reciprocal of the second fraction, and change the expression to multiplication:

$$\frac{3}{6} \times \frac{16}{1}$$

Now we multiply the numerators together, and the denominators together:

$$\frac{3}{6} \times \frac{16}{1} = \frac{48}{6}$$

Now we simplify our answer. 48 is divisible by 6, so:

$$\frac{48}{6} = 48 \div 6 = 8$$

Leah can paint 8 birdhouses.