




HSC COLLEGE  
HSC Specialist Since 2000


# M YEAR 12 MATHEMATICS

## EXTENSION 2 COMPLEX NUMBERS

---

 Level 5, Suite 7 / 11 The Boulevard, Strathfield 2135

 (02) 8012 8488

 [enquiry@skyedu-hsc.com](mailto:enquiry@skyedu-hsc.com)



# Table of Contents

<b>Chapter 1. Introduction to Complex Numbers</b> .....	5
<b>1.1 Arithmetic of Complex Numbers</b> .....	6
<b>Introduction</b> .....	6
<b>The Number System</b> .....	6
<b>Real and Imaginary Numbers</b> .....	7
<b>General Form of a Complex Number</b> .....	7
<b>Purely Real and Purely Imaginary</b> .....	7
<b>Solving a Quadratic Equation (With Real Coefficients)</b> .....	8
<b>Get to know ‘i’</b> .....	8
<b>Operations on Complex Numbers</b> .....	10
<b>Complex Conjugate</b> .....	11
<b>Realising the Denominator</b> .....	13
<b>Condition for <math>z_1 = a + ib</math> and <math>z_2 = c + id</math> to be Equal</b> .....	14
<b>Some Useful Results</b> .....	16
<b>Finding the Square Roots of a Complex Number</b> .....	18
<b>Homework Exercise 1A</b> .....	21
<b>1.2 Geometric Representation of a Complex Number</b> .....	23
<b>Argand Diagram</b> .....	23
<b>Modulus and Argument</b> .....	23
<b>Principal Argument</b> .....	24
<b>Cartesian Form and Modulus-Argument Form</b> .....	24
<b>Converting Cartesian Form into Polar Form</b> .....	25
<b>Conjugate of a Complex Number in Polar Form</b> .....	27
<b>Product and Quotient in Polar Form</b> .....	28
<b>Introduction to De Moivre’s Theorem</b> .....	33
<b>Other Properties of Complex Numbers</b> .....	36
<b>Summary (Properties of Complex Numbers)</b> .....	39



Homework Exercise 1B .....	39
<b>1.3 Other Representations of Complex Numbers .....</b>	<b>43</b>
Euler's Formula .....	43
Products and Quotients in Exponential Form .....	44
Converting between Cartesian, Polar and Exponential Forms .....	45
Homework Exercise 1C .....	49
<b>Chapter 2. Geometrical Representation of Complex Numbers as Vectors .....</b>	<b>50</b>
<b>2.1 Introduction .....</b>	<b>51</b>
<b>2.2 Equivalent Vectors .....</b>	<b>51</b>
<b>2.3 Vector Addition and Subtraction .....</b>	<b>52</b>
Addition and Subtraction .....	52
Head–Tail .....	53
Negative Vectors .....	53
<b>2.4 Vector Rotation .....</b>	<b>54</b>
Rotation through $\theta^\circ$ anticlockwise direction and clockwise direction .....	54
Rotation through $90^\circ$ anticlockwise direction and clockwise direction .....	54
<b>2.5 Multiplication by a Constant Number .....</b>	<b>55</b>
Homework Exercise 2A .....	66
<b>Chapter 3 Geometrical Implications of Complex Numbers .....</b>	<b>70</b>
<b>3.1 Locus Related to the Modulus .....</b>	<b>71</b>
Geometrical Meaning of $ z $ .....	71
Geometrical Description of $ z  = r$ .....	71
Geometrical Meaning of $ z - z_0 $ .....	73
Geometrical Description of $ z - z_0  = r$ .....	73
Geometrical Meaning of $ z - z_1  =  z - z_2 $ .....	75
Geometrical Description of $ z - z_1  =  z - z_2 $ .....	75
<b>3.2 Locus Related to the Argument .....</b>	<b>78</b>
Geometrical Meaning of $\arg z$ .....	78



Geometrical Description of $\arg z = \theta$ .....	78
Geometrical Meaning of $\arg(z - z_0)$ .....	80
Geometrical Description of $\arg(z - z_0) = \theta$ .....	80
<b>3.3 Locus with Other Forms</b> .....	<b>82</b>
<b>Homework Exercise 3A</b> .....	<b>85</b>
<b>3.4 Applications of Locus Problems</b> .....	<b>86</b>
<b>Maximum and Minimum Modulus and Argument</b> .....	<b>86</b>
<b>Locus Problems by Algebraic Method</b> .....	<b>90</b>
<b>The Locus of <math>\arg\left(\frac{z-z_1}{z-z_2}\right) = \theta</math> (Not in HSC Syllabus)</b> .....	<b>94</b>
<b>Homework Exercise 3B</b> .....	<b>97</b>
<b>Chapter 4. Using Complex Numbers</b> .....	<b>99</b>
<b>4.1 Conjugate Root Theorem</b> .....	<b>100</b>
<b>4.2 Solving Equations with Complex Numbers</b> .....	<b>105</b>
<b>Finding the <math>n^{\text{th}}</math> Roots of a Complex Number</b> .....	<b>105</b>
<b>Finding the <math>n^{\text{th}}</math> Roots of Unity</b> .....	<b>108</b>
<b>Homework Exercise 4A</b> .....	<b>116</b>
<b>4.3 De Moivre's Theorem</b> .....	<b>119</b>
<b>Proving De Moivre's Theorem by Mathematical Induction</b> .....	<b>119</b>
<b>Use of De Moivre's Theorem</b> .....	<b>120</b>
<b>Trigonometric Applications</b> .....	<b>121</b>
<b>Homework Exercise 4B</b> .....	<b>128</b>



# Chapter 1. Introduction to Complex Numbers



# 1.1 Arithmetic of Complex Numbers

## Introduction

Consider a quadratic equation  $x^2 + 2x + 3 = 0$

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (Q.F) \\ &= \frac{-2 \pm \sqrt{4 - 4(1)(3)}}{2} \\ &= \frac{-2 \pm \sqrt{-8}}{2} \end{aligned}$$

We cannot go further because there is a negative number inside the square root. In order to solve such a quadratic equation completely, we need the negative square.

$$i = \sqrt{-1} \quad \text{or} \quad i^2 = -1$$

And this negative square root  $i$  is called \_\_\_\_\_

Hence, the solutions to the quadratic equation above are,

$$\begin{aligned} x &= \frac{-2 \pm}{2} \\ &= \frac{-2 \pm}{2} \\ &= \end{aligned}$$

And these roots are called \_\_\_\_\_

## The Number System

