Summer Work -
Honors Algebra to Honors Geometry
Summer 2023

Questions?
Email: mstirling@hollandhall.org
Combining Like Terms
Example: Simplify
\[ 8x^2 + 16xy - 3x^2 + 3xy - 3x \]
\[ 8x^2 - 3x^2 + 16xy + 3xy - 3x \]
\[ 5x^2 - 3x + 19xy \]
Identify &/or Group Like Terms

\[
\begin{array}{ccc}
6x + 11y - 4x + y & -3p - 4t - 5t - 2p & 3x^2y - 5xy^2 + 6x^2y \\
-5m + 3q + 4m - q & 9x - 22y + 18x - 3y & 5x^2 + 2xy - 7x^2 + xy \\
\end{array}
\]

Solving Equations with variables on both sides
Example: Solve
\[ 6a - 12 = 5a + 9 \]
\[ a - 12 = 9 \]
subtract 5a from both sides
Solve each equation
\[ a = 21 \]
add 12 to both sides

\[
\begin{array}{ccc}
3x + 5 = 2x + 11 & 8m + 1 = 7m - 9 & 11q - 6 = 3q + 8q \\
-14 + 3a = 10 - a & -2t + 10 = -t & -7x + 7 = 2x - 11 \\
\end{array}
\]

Literal Equations
solve \( 2p = kx - q \) for \( x \)
\[ 2p = kx - q \]
\[ 2p + q = kx \]
\[ \frac{2p + q}{k} = x \]
Solve for \( x \) by isolating the variable \( x \)
Add \( q \) to each side
Divide each side by \( k \)

Solve for the indicated variable.
\[ ax - c = b; \text{ solve for } x \]
\[ 2x + 4y = 8; \text{ solve for } y \]
\[ \frac{3}{2}y + 4x = -2; \text{ solve for } y \]
Solving Inequalities & Graphing

Example: Solve. & Graph

\[
\begin{align*}
5x - 4 & \geq 4x + 6 \\
x - 4 & \geq 6 \\
x & \geq 10
\end{align*}
\]

\[
\begin{align*}
10 - 7x & < 24 \\
-7x & < 14 \\
x & > -2
\end{align*}
\]

Closed circle at 10; arrow going to the right (greater)

Open circle at -2; arrow going to the right (greater)

Solve & Graph.

<table>
<thead>
<tr>
<th>-x + 2 &gt; 7</th>
<th>-5 + m ≤ 4</th>
<th>z + 6 &gt; -2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-16 - 8x ≥ 0</td>
<td>x - 5 &lt; 4</td>
<td>-3x + 4 ≤ -5</td>
</tr>
<tr>
<td>8x - 6 ≥ 10</td>
<td>9(2x - 5) - 3 &lt; 7x - 4</td>
<td>9x - 11 &gt; 6x - 9</td>
</tr>
</tbody>
</table>
Calculating Slope

Example: Find the slope of a line passing through (3, -9) and (2, -1).

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Formula for slope

\[
m = \frac{-1 - (-9)}{2 - 3} = \frac{-1 + 9}{-1} = -8
\]

Substitute values and simplify

Find slope.

<table>
<thead>
<tr>
<th>(4, 1)</th>
<th>(3, 6)</th>
<th>(5, 6)</th>
<th>(9, 8)</th>
<th>(-1, 7)</th>
<th>(-3, 18)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5, 6)</td>
<td>(9, 8)</td>
<td>(3, 6)</td>
<td>(8, 1)</td>
<td>(9, 8)</td>
<td>(1, 7)</td>
</tr>
<tr>
<td>(5, 6)</td>
<td>(9, 8)</td>
<td>(3, 6)</td>
<td>(5, 6)</td>
<td>(9, 8)</td>
<td>(1, 7)</td>
</tr>
<tr>
<td>(5, 6)</td>
<td>(9, 8)</td>
<td>(3, 6)</td>
<td>(5, 6)</td>
<td>(9, 8)</td>
<td>(1, 7)</td>
</tr>
</tbody>
</table>

Finding the equation of a line (given a point and y-intercept)

Example Find an equation of the line that passes through the point (3,4) and has a y-intercept of 5

\[
y = mx + b
\]

Slope-intercept form

\[
4 = 3m + 5
\]

Substitute 5 for b, 3 for x, and 4 for y.

\[
-1 = 3m
\]

Subtract 5 from both sides

\[
-\frac{1}{3} = m
\]

Divide each side by 5

Find the equation of the line.

\[
y = -\frac{1}{3} x + 5
\]

With slope \(-\frac{1}{3}\) & y-intercept of 5, this is the line’s equation

<table>
<thead>
<tr>
<th>(2,1); b = 5</th>
<th>(7, 0); b = 13</th>
<th>(-5, 3); b = -12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3, -3); b = -2</td>
<td>(-3, 10); b = 8</td>
<td>(-1, 4); b = -8</td>
</tr>
</tbody>
</table>
Finding the equation of a line (given a point and the slope)

Example Find an equation of the line that passes through the point (1,2) and has a slope of -3

\[ y - y_1 = m(x - x_1) \]
\[ y - 2 = -3(x - 1) \]
\[ y - 2 = -3x + 3 \]
\[ y = -3x + 3 \]

Find the equation of the line in slope-intercept form.

| (2,3); m = -4 | (-1, 5); m = 2 | (4, 6); m = ½ |
| (2,3); m = -4 | (-1, 5); m = 2 | (4, 6); m = ½ |
| (-3, -4); m = ¾ | (0, 4); m = -3/2 | (5, 0); m = -4 |

Finding the equation of a line (given two points)

Example Write an equation of the line that passes through the points (4,8) and (3,1).

\[ m = \frac{1-8}{3-4} \]
\[ m = \frac{-7}{-1} = 7 \]
\[ y-1 = 7(x - 3) \]
\[ y=1 = 7x -21 \]
\[ y = 7x - 20 \]

Find the equation of the line in slope-intercept form.

| (6, -3) (1, 2) | (5, -1) (4, -5) | (-3, -7) (0, 8) |
| (6, -3) (1, 2) | (5, -1) (4, -5) | (-3, -7) (0, 8) |
| (-7, 9) (-5, 3) | (-2, 4) (3, -6) | (1, 2) (-1, -4) |
**Standard Form of a Line (Ax + By = C)**

**Example**

Graph a line in standard form $2x + 3y = 6$.

<table>
<thead>
<tr>
<th>Option 1:</th>
<th>Option 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change to slope-intercept form</td>
<td>Keep in Standard form</td>
</tr>
</tbody>
</table>

- $2x + 3y = 6$
- $3y = -2x + 6$
- $y = -\frac{2}{3}x + 2$
- $m = -\frac{2}{3}$ and $b = 2$

- $2x + 3y = 6$
- $m = -\frac{A}{B} = -\frac{2}{3}$
- $b = \frac{C}{B} = \frac{6}{3} = 2$

**Graph the y-intercept of 2, then go down 2 and right 3 to find another point on the line.**

**You can also go up 2 and left 3; connect the points to make a line.**

Graph the equation of each line.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x + 5y = 10$</td>
<td><img src="image1" alt="Graph" /></td>
</tr>
<tr>
<td>$x - 4y = 8$</td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>$2x - 3y = 5$</td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td>$3x - 4y = -12$</td>
<td><img src="image4" alt="Graph" /></td>
</tr>
</tbody>
</table>
**Solving Systems of Equations** (by graphing or substitution)

Example: Solve the system \( y = 2x + 5 \) and \( y = -\frac{1}{2}x - 4 \)

**By Graphing**

Graph \( y = 2x + 3 \)
- y-intercept: 3
- slope: 2

Graph \( y = -\frac{1}{2}x - 7 \)
- y-intercept: -7
- slope: \(-\frac{1}{2}\)

Coordinates for solution: \((-4, -5)\)

**By Substitution**

Given \( y = 2x + 3 \) & \( y = -\frac{1}{2}x - 7 \)

Substitute in place of \( y \)
\[
2x + 3 = -\frac{1}{2}x - 7
\]
Add \( \frac{1}{2}x \) to each side
\[
2\frac{1}{2}x + 3 = -7
\]
Subtract 3 from each side
\[
x = -4
\]
Divide each side by \( 2\frac{1}{2} \)
\[
-10
\]
Substitute \((-4)\) in place of \( x \)
\[
y = 2(-4) + 3
\]
Simplify
\[
y = -8 + 3
\]
Combine like terms to find \( y \)
\[
y = -5
\]
Coordinates for solution: \((-4, -5)\)

Solve each system by graphing or substitution:

<table>
<thead>
<tr>
<th>System</th>
<th>By Graphing</th>
<th>By Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 2x + 4 ) &lt;br&gt;(-3x + y = -9 )</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>( y = x - 1 ) &lt;br&gt;( x + y = 3 )</td>
<td></td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
<tr>
<td>( 4x + y = 0 ) &lt;br&gt;( x + 2y = -7 )</td>
<td><img src="image4.png" alt="Graph" /></td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2}x + 2y = 12 ) &lt;br&gt;( x - 2y = 6 )</td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
Solve Systems of Equations (by elimination)

Example

Given

Multiply by 2 to eliminate the y

Add the 2 equation

Divide each side by 4

Given

Multiply by 2 to eliminate the y

Add the 2 equation

Divide each side by 4

Substitute 0 for x in

either equation; simplify

Divide each side by 2

Solution to system

Solve each system by elimination.

| x - y = 1 | 3x + 4y = 19 | 2x - 3y = 9 |
| x + y = 3 | 3x + 6y = 33 | -5x - 3y = 30 |

| 5x - y = -6 | 6x - 3y = 6 | The sum of two numbers is 28. Their difference is 4. What are the two numbers? |
| -x + y = 2 | 6x + 8y = -16 | |

The sum of two numbers is 28. Their difference is 4. What are the two numbers?
Solving Proportions

Example

\[
\frac{x}{8} = \frac{3}{4}
\]

\[
4x = 24 \quad \text{Cross multiply}
\]

\[
\frac{6}{x + 4} = \frac{2}{9}
\]

\[
6 \cdot 9 = 2(x + 4) \quad \text{Cross Multiply}
\]

\[
x = 6
\]

\[
46 = 2x \quad \text{Subtract 8 from both sides}
\]

\[
x = 23 \quad \text{Divide each side by 2}
\]

Solve each proportion to find the value of the given variable.

<table>
<thead>
<tr>
<th>( \frac{y}{40} = \frac{3}{8} )</th>
<th>( \frac{3}{p - 6} = \frac{1}{p} )</th>
<th>( \frac{3}{8} = \frac{3}{2d} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{r}{3r + 1} = \frac{2}{3} )</td>
<td>( \frac{3}{m + 4} = \frac{9}{14} )</td>
<td>( \frac{w}{4} = \frac{9}{w} )</td>
</tr>
</tbody>
</table>

Property of Exponents

<table>
<thead>
<tr>
<th>Property</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product of Powers</td>
<td>( a^m \cdot a^n = a^{m+n} )</td>
</tr>
<tr>
<td>Power of a Power</td>
<td>((a^m)^n = a^{m \cdot n})</td>
</tr>
<tr>
<td>Power of a Product</td>
<td>((ab)^m = a^m b^m)</td>
</tr>
<tr>
<td>Negative Power</td>
<td>(a^{-n} = \frac{1}{a^n})</td>
</tr>
<tr>
<td>Zero Power</td>
<td>(a^0 = 1)</td>
</tr>
<tr>
<td>Quotient of Powers</td>
<td>(\frac{a^m}{a^n} = a^{m-n})</td>
</tr>
<tr>
<td>Power of Quotients</td>
<td>(\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m})</td>
</tr>
</tbody>
</table>
Simplify each exponent. Answers should be written using positive exponents.

<table>
<thead>
<tr>
<th>$g^5 \cdot g^{11}$</th>
<th>$(b^5)^3$</th>
<th>$w^7$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\frac{y^{12}}{y^n}$</th>
<th>$(3x^3)(-5x^2)$</th>
<th>$(-4a^5b^0c)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$-6(x^2y^3)^4$</th>
<th>$(-18mn^4)(-\frac{3}{4}mn^2)$</th>
<th>$\frac{16x^5y^3}{2x^3y^3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Polynomial Operations**

<table>
<thead>
<tr>
<th>Add or Subtract like terms</th>
<th>Distributing</th>
<th>Multiplying binomials &amp;/or trinomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(7x^2 + 4x - 3) - (-5x^2 - 3x + 2)$</td>
<td>-2x(5x + 11)</td>
<td>$(7x - 3)(3x + 7)$ <em>Multiply everything in the 1st binomial times the 2nd</em></td>
</tr>
<tr>
<td>$7x^2 - (5x^2) + 4x - (-3x) - 3(2)$</td>
<td>-10$x^2 - 22x$</td>
<td>$7x - 3x + 7x - 7 - 3x - 3 - 7$ <em>First</em> - <em>Outside</em> - <em>Inside</em> - <em>Last</em> (FOIL)</td>
</tr>
<tr>
<td>$12x^2 + 7x - 5$</td>
<td></td>
<td>$21x^2 + 49x - 9x - 21$ <em>Simplify</em></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$21x^2 + 40x - 21$ <em>Combine like terms</em></td>
</tr>
</tbody>
</table>

Simplify each polynomial

<table>
<thead>
<tr>
<th>$(2x + 3y) + (4x + 9y)$</th>
<th>$(7x^2 + x + 1) - (3x^2 - 4x - 3)$</th>
<th>$(7a^2 - a + 4) - (3a^2 - 4a - 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$-3x(8x^2 - 3x + 1)$</th>
<th>$-10pq(3pq + 4p - 5q^2)$</th>
<th>$5w(w^2 - 7w + 3) - 2w(2w^2 - 5w + 2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$(x + 4)(x - 7)$</th>
<th>$(5x - 2y)(3x + 9y)$</th>
<th>$(2 + 5)(4z - 6)$</th>
</tr>
</thead>
</table>
Factoring Polynomials \((ax^2 + bx + c)\)

Examples:

<table>
<thead>
<tr>
<th>Factoring out GCF</th>
<th>Difference of squares</th>
<th>Perfect Square Trinomials</th>
<th>Trinomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6x^2 + 21x)</td>
<td>(x^2 - 64)</td>
<td>(4x^2 + 12x + 9)</td>
<td>(3x^2 + 7x + 2)</td>
</tr>
<tr>
<td>(3x(2x + 7))</td>
<td>((x + 8)(x - 8))</td>
<td>((2x + 3)^2)</td>
<td>((3x + 1)(x + 2))</td>
</tr>
</tbody>
</table>

Factor completely.

<table>
<thead>
<tr>
<th>(6e^3f - 11ef)</th>
<th>(y^2 - 5y - 84)</th>
<th>(6x^2 + 7x + 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6z^2 - 5z - 4)</td>
<td>(75x^2 - 147y^2)</td>
<td>(x^2 - 25)</td>
</tr>
<tr>
<td>(x^2 - 6x + 9)</td>
<td>(16c^2 + 72cd + 81d^2)</td>
<td>(x^4 - 16)</td>
</tr>
</tbody>
</table>

Solving Quadratics

Example \(x^2 + 3x^2 = 10\)

\(x^2 + 3x^2 - 10 = 0\)

\((x - 5)(x + 2) = 0\)

\(x - 5 = 0\) and \(x + 2 = 0\)

\(x = 5\) and \(x = -2\)

Subtract 10 from both sides so the quadratic is equal to 0

Factor (see previous section on different kinds of factoring)

Use Zero Product Property and put each binomial equal to 0

Solve both for \(x\).

Solve each quadratic to find the possible values of \(x\).

<table>
<thead>
<tr>
<th>(3x^2 - 12 = 0)</th>
<th>(6x^2 - 5x + 1 = 0)</th>
<th>(x^2 + 7x = 18)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^2 + 11x = 80)</td>
<td>(2x^2 = x + 15)</td>
<td>(3x^3 + 3x^2 - 60x = 0)</td>
</tr>
</tbody>
</table>
Simplifying Radicals

An expression is in simplest radical form when:
1) there is no integer under the radical sign with a perfect square
2) there are no fractions under the radical sign
3) there are no radical in the denominator

Examples:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{20} )</td>
<td>( \sqrt{4} \cdot \sqrt{5} )</td>
</tr>
<tr>
<td>( \sqrt{4 \cdot \sqrt{5}} )</td>
<td>( \sqrt{13} \cdot \sqrt{7} )</td>
</tr>
<tr>
<td>( 2\sqrt{5} )</td>
<td>( \sqrt{\frac{49}{9}} )</td>
</tr>
</tbody>
</table>

Express the following in simplest radical form.

| \( \sqrt{121} \) | \( \sqrt{40} \) | \( \sqrt{72} \) |
| \( \sqrt{\frac{25}{36}} \) | \( \sqrt{\frac{27}{45}} \) | \( \sqrt{\frac{50}{75}} \) |
| \( \frac{\sqrt{32}}{\sqrt{5}} \) | \( \sqrt{320} \) | \( \sqrt{5} \cdot \sqrt{60} \) |
**Pythagorean Theorem**

Example: If $c$ is the measure of the hypotenuse of a right triangle, find each missing measure.

$a = 6, c = 10, b = ____$

\begin{align*}
6^2 + b^2 &= 10^2 \\
36 + b^2 &= 100 \\
b^2 &= 64 \\
b &= 8
\end{align*}

If $c$ is the measure of the hypotenuse of a right triangle, find each missing measure.

<table>
<thead>
<tr>
<th>$a$ = 5, $b$ = 12, $c$ = _____</th>
<th>$a$ = 20, $b$ = _____ &amp; $c$ = 29</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = \sqrt{6}, b = \sqrt{19}, c = _____$</td>
<td>$a = _____, b = 12, &amp; c = 15$</td>
</tr>
<tr>
<td>$a = 8, b = 15, c = _____$</td>
<td>$a = _____, b = 40, &amp; c = 41$</td>
</tr>
</tbody>
</table>