## Summer Work -

Honors Algebra to Honors Geometry
Summer 2023

Questions?
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## Combining Like Terms

Example:Simplify

$$
\begin{aligned}
& 8 x^{2}+16 x y-3 x^{2}+3 x y-3 x \\
& 8 x^{2}-3 x^{2}+16 x y+3 x y-3 x \\
& 5 x^{2}-3 x+19 x y
\end{aligned} \quad \text { Identify \&/or Group Like Terms }
$$

Simplify

| $6 x+11 y-4 x+y$ | $-3 p-4 t-5 t-2 p$ | $3 x^{2} y-5 x y^{y}+6 x^{2} y$ |
| :---: | :---: | :---: |
| $-5 m+3 q+4 m-q$ | $9 x-22 y+18 x-3 y$ | $5 x^{2}+2 x y-7 x^{2}+x y$ |

Solving Equations with variables on both sides

| Example: Solve | $6 \mathrm{a}-12=5 a$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{a}-12=9$ | subtract 5a |  |
| Solve each equation | $\mathrm{a}=21$ | add 12 to bo |  |
| $3 x+5=2 x+11$ |  | $8 m+1=7 m-9$ | $11 q-6=3 q+8 q$ |
| $-14+3 \mathrm{a}=10-\mathrm{a}$ |  | $-2 t+10=-t$ | $-7 x+7=2 x-11$ |

## Literal Equations



## Solving Inequalities \& Graphing



Solve \& Graph.

| $-x+2>7$ | $-5+m \leq 4$ | $z+6>-2$ |
| :---: | :---: | :---: |
| $-16-8 x \geq 0$ | $x-5<4$ | $-3 x+4 \leq-5$ |
| $8 x-6 \geq 10$ | $9(2 x-5)-3<7 x-4$ | $9 x-11>6 x-9$ |
|  |  |  |

## Calculating Slope

Example: $\quad$ Find the slope of a line passing through $(3,-9)$ and $(2,-1)$.

| Find slope. | $\begin{aligned} & m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\ & m=\frac{-1-(-9)}{2-3}=\frac{-1+9}{-1} \\ & m=\frac{8}{-1}=-8 \end{aligned}$ | Formula for slope <br> Substitute values and simplify <br> Slope is -8 |
| :---: | :---: | :---: |
| $(4,1)(3,6)$ | $(5,6)(9,8)$ | $(-1,7)(-3,18)$ |
| $(-8,0)(5,-2)$ | $(0,-4)(7,3)$ | $(-6,-4)(1,10)$ |

Finding the equation of a line (given a point and $y$-intercept)
Example Find an equation of the line that passes through the point $(3,4)$ and has a $y$-intercept of 5

|  | $y=m x+b$ | Slope-intercept form |
| :---: | :---: | :---: |
|  | $4=3 m+5$ | Substitute 5 for b, 3 for $x$, and 4 for $y$. |
|  | $-1=3 \mathrm{~m}$ | Subtract 5 from both sides |
|  | $-1 / 3=m$ | Divide each side by 5 |
| Find the equation of the line. | $y=-1 / 3 x+5$ | With slope $-1 / 3$ \& $y$-intercept of 5 , this is the line's equation |
| $(2,1) ; \mathrm{b}=5$ | $(7,0) ; b=13$ | $(-5,3) ; b=-12$ |
| $(-3,-3) ; b=-2$ | $(-3,10) ;$ b $=8$ | $(-1,4) ; b=-8$ |

Finding the equation of a line (given a point and the slope)
Example Find an equation of the line that passes through the point $(1,2)$ and has a slope of -3

$$
\begin{array}{ll}
y-y_{1}=m\left(x-x_{1}\right) & \text { Point-Slope Form } \\
y-2=-3(x-1) & \text { Substitute } 2 \text { for } y_{1} 1 \text { for } x_{1} \text { and }-3 \text { for } m . \\
y-2=-3 x+3 & \text { Distribute the }-2 \\
y=-3 x+5 & \text { Add } 2 \text { to both sides }
\end{array}
$$

Find the equation of the line in slope-intercept form.

| $(2,3) ; \mathrm{m}=-4$ | $(-1,5) ; \mathrm{m}=2$ | $(4,6) ; \mathrm{m}=-1 / 2$ |
| :--- | :--- | :--- |
| $(-3,-4) ; \mathrm{m}=2 / 3$ | $(0,4) ; \mathrm{m}=-3 / 2$ | $(5,0) ; \mathrm{m}=-4$ |

Finding the equation of a line (given two points)
Example $\quad$ Write an equation of the line that passes through the points $(4,8)$ and $(3,1)$.

$$
\begin{array}{ll}
m=\frac{1-8}{3-4} & \text { Substitute values into the formula for slope } \\
m=\frac{-7}{-1}=7 & \text { Simplify } \\
y-1=7(x-3) & \text { Select either point and substitute values of point and slope into point-slope form } \\
y-1=7 x-21 & \text { Distribute the } 7 \\
y=7 x-20 & \text { Add } 1 \text { to each side to get the equation of a line in slope-intercept form }
\end{array}
$$

Find the equation of the line in slope-intercept form.

| $(6,-3)(1,2)$ | $(5,-1)(4,-5)$ | $(-3,-7)(0,8)$ |
| :--- | :--- | :--- |
| $(-7,9)(-5,3)$ | $(-2,4)(3,-6)$ | $(1,2)(-1,-4)$ |

## Standard Form of a Line ( $\mathrm{Ax}+\mathrm{By}=\mathrm{C}$ )

| Example | Graph a line in standard form $2 x+3 y=6$ |  |  |
| :--- | :--- | :--- | :--- |
| Option 1: | Change to slope-intercept form | Option 2 | Keep in Standard form |
| $2 x+3 y=6$ |  | $2 x+3 y=6$ |  |
| $3 y=-2 x+6$ | Subtract $2 x$ from each side | $m=\frac{-A}{B}=\frac{-2}{3}$ | Slope in standard form $\frac{-A}{B}$ |
| $y=-2 / 3 x+2$ | Divide everything by 3 |  |  |
| $m=-2 / 3$ and $b=2$ | Graph ay-intercept at $2 \& a$ | $b=\frac{C}{B}=\frac{6}{3}=2$ | y-intercept in stand form $b=\frac{C}{B}$ |
| slope of- $-2 / 3$ |  |  |  |



Graph the $y$-intercept of 2, then go down 2 and right 3 to find another point on the line.
You can also go up 2 and left 3; connect the points to make a line.

Graph the equation of each ${ }^{\prime}$ line.

| $4 x+5 y=10$ |  | $x-4 y=8$ |  |
| :---: | :---: | :---: | :---: |
| $2 x-3 y=5$ |  | $3 x-4 y=-12$ |  |

Solving Systems of Equations (by graphing or substitution )

| Example Solve the system $y=2 x+5$ and $y=-1 / 2 x-4$ |  |  |  |
| :---: | :---: | :---: | :---: |
| By Graphing |  | By substitution |  |
| Graph $\mathrm{y}=2 \mathrm{x}+3$ | rif | Given $\mathrm{y}=2 \mathrm{x}+3$ \& $\mathrm{y}=-1 / 2 \mathrm{x}-7$ |  |
| $y$-intercept of 3 |  | $2 x+3=-1 / 2 x-7$ | Substitute in place of $y$ |
| slope of 2 |  | $21 / 2 x+3=-7$ | Add $1 / 2 \times$ to each side |
|  |  | $21 / 2 x=-10$ | Subtract 3 from each side |
| Graph $\mathrm{y}=-1 / 2 \mathrm{x}-7$ | - | $x=-4$ | Divide each side by $21 / 2$ |
| $y$-intercept of -7 |  |  |  |
| slope of $-1 / 2$ | $1 /$ | $y=2(-4)+3$ | Substitute (-4) in place of $x$ |
|  |  | $y=-8+3$ | Simplify |
| $(-4,-5)$ Coordinates for solution |  | $y=-5$ | Combine like terms to find $y$ |
|  |  | (-4, -5) | Coordinates for solution |

Solve each system by graphing or substitution

| $\begin{aligned} & y=2 x+4 \\ & -3 x+y=-9 \end{aligned}$ |  | $\begin{aligned} & y=x-1 \\ & x+y=3 \end{aligned}$ |
| :---: | :---: | :---: |
| $\begin{aligned} & 4 x+y=0 \\ & x+2 y=-7 \end{aligned}$ |  | $\begin{aligned} & 1 / 2 x+2 y=12 \\ & x-2 y=6 \end{aligned}$ |

Solve Systems of Equations (by elimination)

Example


| $2(0)+2 y=-4$ | Substitute 0 for $x$ in |
| :--- | :--- |
| $2 y=-4$ | either equation; simplify |
| $y=-2$ | Divide each side by 2 |

$$
(0,-2) \quad \text { Solution to system }
$$

Solve each system by elimination.


## Solving Proportions

Example

$$
\begin{aligned}
\frac{x}{8} & =\frac{3}{4} \\
4 x & =8 \cdot 3 \\
4 x & =24 \\
x & =6
\end{aligned} \quad \text { Cross multiply }
$$

$$
\frac{6}{x+4}=\frac{2}{9}
$$

$$
6 \cdot 9=2(x+4) \quad \text { Cross Multiply }
$$

$$
54=2 x+8 \quad \text { Simplify both sides }
$$

$$
46=2 x \quad \text { Subtract } 8 \text { from both sides }
$$

$$
x=23 \quad \text { Divide each side by } 2
$$

Solve each proportion to find the value of the given variable.

| $\frac{y}{40}=\frac{3}{8}$ | $\frac{3}{p-6}=\frac{1}{p}$ | $\frac{3}{8}=\frac{3}{2 d}$ |
| :--- | :--- | :--- |
| $\frac{r}{3 r+1}=\frac{2}{3}$ | $\frac{3}{m+4}=\frac{9}{14}$ | $\frac{w}{4}=\frac{9}{w}$ |

## Property of Exponents

| Property |  | Example |
| :--- | :--- | :--- |
| Product of Powers | $a^{m} \cdot a^{n}=a^{m+n}$ | $x^{4} \cdot x^{2}=x^{6}$ |
| Power of a Power | $\left(a^{m}\right)^{n}=a^{m \cdot n}$ | $\left(x^{4}\right)^{2}=x^{8}$ |
| Power of a Product | $(a b)^{m}=a^{m} b^{m}$ | $(2 x)^{3}=8 x^{3}$ |
| Negative Power | $a^{-n}=\frac{1}{a^{n}}$ | $x^{-3}=\frac{1}{x^{3}}$ |
| Zero Power | $a^{0}=1$ | $4^{0}=1$ |
| Quotient of Powers | $\frac{a^{m}}{a^{n}}=a^{m-n}$ | $\frac{x^{10}}{x^{4}}=x^{6}$ |
| Power of Quotients | $\left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}$ | $\left(\frac{x}{y}\right)^{3}=\frac{a^{3}}{b^{3}}$ |

Simplify each exponent. Answers should be written using positive exponents.

| $g^{5} \cdot g^{11}$ | $\left(b^{6}\right)^{3}$ | $w^{-7}$ |
| :---: | :---: | :---: |
| $\frac{y^{12}}{y^{8}}$ | $\left(3 x^{7}\right)\left(-5 x^{3}\right)$ | $\left(-4 a^{5} b^{0} c\right)^{2}$ |
| $-6\left(x^{2} y^{3}\right)^{4}$ | $\left(-18 m n^{4}\right)\left(-1 / 6 \mathrm{mn}^{2}\right)$ | $\frac{16 x^{5} y^{2}}{2 x^{3} y^{3}}$ |

## Polynomial Operations

| Add or Subtract like terms | Distributing | Multiplying binomials \&/or trinomials |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| $\left(7 x^{2}+4 x-3\right)-\left(-5 x^{2}-3 x+2\right)$ | $-2 x(5 x+11)$ | $(7 x-3)(3 x+7)$ | Multiply everything in the $1^{\text {st }}$ binomial times the 2 |

Simplify each polynomial

| $(2 x+3 y)+(4 x+9 y)$ | $\left(7 x^{2}+x+1\right)-\left(3 x^{2}-4 x-3\right)$ | $\left(7 a^{2}-a+4\right)-\left(3 a^{2}-4 a-3\right)$ |
| :---: | :---: | :---: |
| $-3 x\left(8 x^{2}-3 x+1\right)$ | $-10 p q\left(3 p q+4 p-5 q^{2}\right)$ | $5 w\left(w^{2}-7 w+3\right)-2 w\left(2 w^{2}-5 w+2\right)$ |
| $(x+4)(x-7)$ | $(5 x-2 y)(3 x+9 y)$ | $(z+5)(4 z-6)$ |

Factoring Polynomials ( $a x^{2}+b x+c$ )
Examples:

| Factoring out GCF | Difference of squares | Perfect Square Trinomials | Trinomials |
| :--- | :--- | :--- | :--- |
| $6 x^{2}+21 x$ | $x^{2}-64$ | $4 x^{2}+12 x+9$ | $3 x^{2}+7 x+2$ |
| $3 x(2 x+7)$ | $(x+8)(x-8)$ | $(2 x+3)^{2}$ | $(3 x+1)(x+2)$ |

Factor completely.

| $6 e^{3} f-11 e f$ | $y^{2}-5 y-84$ | $6 x^{2}+7 x+2$ |
| :---: | :---: | :---: |
| $6 z^{2}-5 z-4$ | $75 x^{2}-147 y^{2}$ | $x^{2}-25$ |
| $x^{2}-6 x+9$ | $16 c^{2}+72 c d+81 d^{2}$ | $x^{4}-16$ |

## Solving Quadratics

Example $\quad x^{2}+3 x^{2}=10$

$$
\begin{array}{cl}
x^{2}+3 x^{2}-10=0 & \text { Subtract } 10 \text { from both sides so the quadratic is equal to } 0 \\
(x-5)(x+2)=0 & \text { Factor (see previous section on different kinds of factoring) } \\
x=5 \text { and } x+2=0 & \text { Use Zero Product Property and put each binomial equal to } 0 \\
x=-2 & \text { Solve both for } x .
\end{array}
$$

Solve each quadratic to find the possible values of x .

| $3 x^{2}-12=0$ | $6 x^{2}-5 x+1=0$ | $x^{2}+7 x=18$ |
| :---: | :---: | :---: |
| $x^{2}+11 x=80$ | $2 x^{2}=x+15$ | $3 x^{3}+3 x^{2}-60 x=0$ |
|  |  |  |

## Simplifying Radicals

## An expression is in simplest radical form when:

1.) there is no integer under the radical sign with a perfect square
2.) there are no fractions under the radical sign
3.) there are no radical in the denominator

Examples:
$\sqrt{20}$
$\sqrt{4} \cdot \sqrt{5}$
$2 \sqrt{5}$
$\begin{array}{ll}\sqrt{\frac{13}{49}} & \sqrt{\frac{9}{24}} \\ \frac{\sqrt{13}}{\sqrt{49}} & \frac{\sqrt{9}}{\sqrt{24}}\end{array}$
$\frac{3}{\sqrt{4} \cdot \sqrt{6}}=\frac{3}{2 \cdot \sqrt{6}}$
$\frac{3}{2 \cdot \sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$
$\frac{3 \sqrt{6}}{2 \sqrt{36}}=\frac{3 \sqrt{6}}{2 \cdot 6}$
$\frac{3 \sqrt{6}}{12}=\frac{\sqrt{6}}{4}$

Express the following in simplest radical form.

| $\sqrt{121}$ | $\sqrt{40}$ | $\sqrt{72}$ |
| :---: | :---: | :---: |
| $\sqrt{\frac{25}{36}}$ | $\sqrt{\frac{27}{45}}$ | $\sqrt{\frac{50}{75}}$ |
| $\frac{\sqrt{32}}{\sqrt{5}}$ |  | $\sqrt{5} \cdot \sqrt{60}$ |
|  |  |  |

## Pythagorean Theorem



Example: If c is the measure of the hypotenuse of a right triangle, find each missing measure.

$$
a=6, c=10, b=
$$

$$
\begin{aligned}
& a^{2}+b^{2}=c^{2} \\
& 6^{2}+b^{2}=10^{2} \\
& 36+b^{2}=100 \\
& b^{2}=64 \\
& b=8
\end{aligned}
$$

If c is the measure of the hypotenuse of a right triangle, find each missing measure.


