Summer Work -

Honors Geometry to Honors Algebra II

Summer 2023

Questions?

Email: mstirling@hollandhall.org

Combining Like Terms

Example: Simplify $8x^2 + 16xy - 3x^2 + 3xy - 3x$

$8x^2 - 3x^2 + 16xy + 3xy - 3x$ *Identify &/or Group Like Terms*

$5x^2 - 3x + 19xy$

Simplify

$6x + 11y - 4x + y$	$-3p - 4t - 5t - 2p$	$3x^2y - 5xy^y + 6x^2y$
$-5m + 3q + 4m - q$	$9x - 22y + 18x - 3y$	$5x^2 + 2xy - 7x^2 + xy$

Solving Equations with variables on both sides

Example: Solve $6a - 12 = 5a + 9$

                                 $a - 12 = 9$ *subtract 5a from both sides*

Solve each equation $a = 21$ *add 12 to both sides*

$3x + 5 = 2x + 11$	$8m + 1 = 7m - 9$	$11q - 6 = 3q + 8q$
$-14 + 3a = 10 - a$	$-2t + 10 = -t$	$-7x + 7 = 2x - 11$

Literal Equations

                 Solve $2p = kx - q$ for x

                       $2p = kx - q$ *Solve for x by isolating the variable x*

                       $2p + q = kx$ *Add q to each side*

                       $\frac{2p + q}{k} = x$ *Divide each side by k*

Solve for the indicated variable. $x = \frac{2p + q}{k}$

$ax - c = b$; solve for x	$2x + 4y = 8$; solve for y	$\frac{2}{3}y + 4x = -2$; solve for y

Solving Inequalities & Graphing

Example: Solve. & Graph

$5x - 4 \geq 4x + 6$ $10 - 7x < 24$

$x - 4 \geq 6$ $-7x < 14$
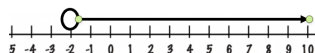
$x \geq 10$ $x > -2$

> When you multiply or divide each side of an inequality by a negative number, you must reverse the inequality symbol to maintain a true statement.
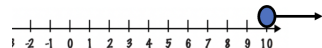
Closed circle at 10; arrow going to the right (greater)

Open circle at -2; arrow going to the right (greater)

Solve & Graph.

$-x + 2 > 7$	$-5 + m \leq 4$	$z + 6 > -2$
$-16 - 8x \geq 0$	$x - 5 < 4$	$-3x + 4 \leq -5$
$8x - 6 \geq 10$	$9(2x - 5) - 3 < 7x - 4$	$9x - 11 > 6x - 9$

Calculating Slope

Example: Find the slope of a line passing through (3, - 9) and (2, -1).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ *Formula for slope*

$$m = \frac{-1-(-9)}{2-3} = \frac{-1+9}{-1}$$ *Substitute values and simplify*

Find slope. $m = \frac{8}{-1} = -8$ *Slope is -8*

(4 , 1) (3, 6)	(5, 6) (9, 8)	(-1, 7) (-3, 18)
(-8, 0) (5, -2)	(0, - 4) (7, 3)	(-6, -4) (1, 10)

Finding the equation of a line (given a point and y-intercept)

Example Find an equation of the line that passes through the point (3,4) and has a y-intercept of 5

y = mx + b   *Slope-intercept form*

4 = 3m + 5   *Substitute 5 for b, 3 for x, and 4 for y.*

-1 = 3m   *Subtract 5 from both sides*

-⅓ = m   *Divide each side by 5*

Find the equation of the line.   y = -⅓ x + 5   *With slope -⅓ & y-intercept of 5, this is the line's equation*

(2,1); b = 5	(7, 0); b = 13	(-5, 3); b = -12
(-3, -3); b = -2	(-3, 10); b = 8	(-1, 4); b = -8

Finding the equation of a line (given a point and the slope)

Example Find an equation of the line that passes through the point (1,2) and has a slope of -3

$$y - y_1 = m(x - x_1)$$ *Point-Slope Form*

$$y - 2 = -3(x - 1)$$ *Substitute 2 for y_1 1 for x_1 and -3 for m.*

$$y - 2 = -3x + 3$$ *Distribute the -2*

$$y = -3x + 5$$ *Add 2 to both sides*

Find the equation of the line in slope-intercept form.

(2,3); m = -4	(-1, 5); m = 2	(4, 6); m = -½
(-3, - 4); m = ⅔	(0, 4); m = -3/2	(5, 0); m = -4

Finding the equation of a line (given two points)

Example   Write an equation of the line that passes through the points (4,8) and (3,1).

$$m = \frac{1-8}{3-4}$$ *Substitute values into the formula for slope*

$$m = \frac{-7}{-1} = 7$$ *Simplify*

$$y - 1 = 7(x - 3)$$ *Select either point and substitute values of point and slope into point-slope form*

$$y - 1 = 7x - 21$$ *Distribute the 7*

$$y = 7x - 20$$ *Add 1 to each side to get the equation of a line in slope-intercept form*

Find the equation of the line in slope-intercept form.

(6, -3) (1, 2)	(5, -1) (4, -5)	(-3, -7) (0, 8)
(-7, 9) (-5, 3)	(-2, 4) (3, -6)	(1, 2) (-1, -4)

Standard Form of a Line (Ax + By = C)

Example   Graph a line in standard form 2x+3y = 6

Option 1:	Change to slope-intercept form	Option 2	Keep in Standard form

Option 1:   Change to slope-intercept form

2x + 3y = 6

3y = -2x + 6   *Subtract 2x from each side*

y = -⅔ x + 2   *Divide everything by 3*

m = -⅔ and b = 2   *Graph a y-intercept at 2 & a slope of -⅔*

Option 2   Keep in Standard form

2x + 3y = 6

$m = \frac{-A}{B} = \frac{-2}{3}$ Slope in standard form $\frac{-A}{B}$

$b = \frac{C}{B} = \frac{6}{3} = 2$ y-intercept in stand form b= $\frac{C}{B}$

Graph using a slope of -⅔ & y-intercept of 2
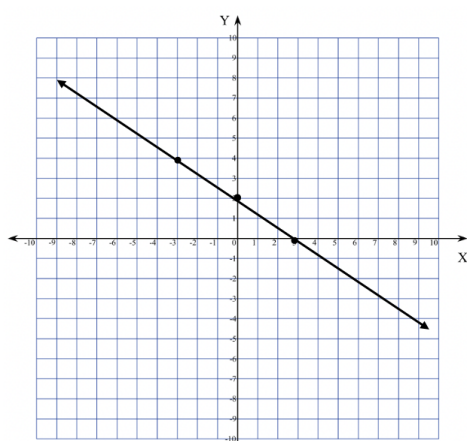
Graph the y-intercept of 2, then go down 2 and right 3 to find another point on the line.
You can also go up 2 and left 3; connect the points to make a line.

Graph the equation of each line.

4x + 5y = 10		x -4y = 8	
2x - 3y = 5		3x - 4y = -12	

Solving Systems of Equations (by graphing or substitution)

Example   Solve the system y = 2x + 5 and y = -½ x -4

By Graphing

Graph y = 2x + 3
y-intercept of 3
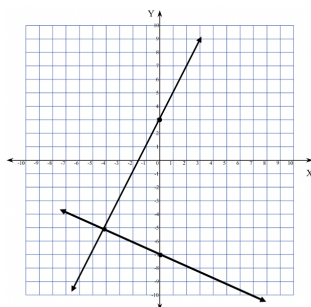slope of 2

Graph y = -½ x - 7
y-intercept of -7
slope of -½

(-4, -5) Coordinates for solution

By substitution

Given y= 2x + 3 & y= -½ x -7

2x + 3 = -½ x - 7	*Substitute in place of y*
2 ½x + 3 = -7	*Add ½ x to each side*
2 ½ x = -10	*Subtract 3 from each side*
x = -4	*Divide each side by 2 ½*
y= 2(-4) + 3	*Substitute (-4) in place of x*
y = -8 + 3	*Simplify*
y = -5	*Combine like terms to find y*
(-4, -5)	*Coordinates for solution*

Solve each system by graphing or substitution

y= 2x + 4 -3x + y = -9		y = x - 1 x + y = 3
4x + y = 0 x + 2y = -7		½ x + 2y = 12 X - 2y = 6

Solve Systems of Equations (by elimination)

Example

$$x - y = 4$$
$$2x + 2y = -4$$

Given

⇒

$$2(x - y = 4)$$
$$2x + 2y = -4$$

Multiply by 2 to eliminate the y

⇒

$$2x - 2y = 8$$
$$\underline{2x + 2y = -8}$$
$$4x \quad = 0$$
$$x = 0$$

Add the 2 equation

Divide each side by 4

$$2(0) + 2y = -4$$
$$2y = -4$$
$$y = -2$$

Substitute 0 for x in

either equation; simplify

Divide each side by 2

$$(0, -2)$$

Solution to system

Solve each system by elimination.

$x - y = 1$ $x + y = 3$	$3x + 4y = 19$ $3x + 6y = 33$	$2x - 3y = 9$ $-5x - 3y = 30$
$5x - y = -6$ $-x + y = 2$	$6x - 3y = 6$ $6x + 8y = -16$	The sum of two numbers is 28. Their difference is 4. What are the two numbers?

Solving Proportions

Example

$$\frac{x}{8} = \frac{3}{4}$$

$4x = 8 \cdot 3$ *Cross multiply*

$4x = 24$

$x = 6$

$$\frac{6}{x+4} = \frac{2}{9}$$

$6 \cdot 9 = 2(x + 4)$ *Cross Multiply*

$54 = 2x + 8$ *Simplify both sides*

$46 = 2x$ *Subtract 8 from both sides*

$x = 23$ *Divide each side by 2*

Solve each proportion to find the value of the given variable.

$\frac{y}{40} = \frac{3}{8}$	$\frac{3}{p-6} = \frac{1}{p}$	$\frac{3}{8} = \frac{3}{2d}$
$\frac{r}{3r+1} = \frac{2}{3}$	$\frac{3}{m+4} = \frac{9}{14}$	$\frac{w}{4} = \frac{9}{w}$

Property of Exponents

Property		Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$x^4 \cdot x^2 = x^6$
Power of a Power	$(a^m)^n = a^{m \cdot n}$	$(x^4)^2 = x^8$
Power of a Product	$(ab)^m = a^m b^m$	$(2x)^3 = 8x^3$
Negative Power	$a^{-n} = \frac{1}{a^n}$	$x^{-3} = \frac{1}{x^3}$
Zero Power	$a^0 = 1$	$4^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^{10}}{x^4} = x^6$
Power of Quotients	$(\frac{a}{b})^m = \frac{a^m}{b^m}$	$(\frac{x}{y})^3 = \frac{a^3}{b^3}$

Simplify each exponent. Answers should be written using positive exponents.

$g^5 \cdot g^{11}$	$(b^6)^3$	w^{-7}
$\dfrac{y^{12}}{y^8}$	$(3x^7)(-5x^3)$	$(-4a^5b^0c)^2$
$-6(x^2y^3)^4$	$(-18mn^4)(-\tfrac{1}{6}\,mn^2)$	$\dfrac{16x^5y^2}{2x^3y^3}$

Polynomial Operations

Add or Subtract like terms	Distributing	Multiplying binomials &/or trinomials
$(7x^2 + 4x - 3) - (-5x^2 - 3x + 2)$ $7x^2 -(-5x^2) + 4x - (-3x) -3-(2)$ $12x^2 + 7x - 5$	$-2x(5x + 11)$ $-10x^2 - 22x$	$(7x - 3)(3x + 7)$ *Multiply everything in the 1st binomial times the 2nd* $7x\cdot3x +7x\cdot7 -3\cdot3x -3\cdot7$ *First-Outside-Inside-Last (FOIL)* $21x^2 + 49x - 9x - 21$ *Simplify* $21x^2 +40x - 21$ *Combine like terms*

Simplify each polynomial

$(2x +3y) + (4x + 9y)$	$(7x^2 + x + 1) - (3x^2 - 4x - 3)$	$(7a^2 - a + 4) - (3a^2 - 4a - 3)$
$-3x(8x^2 - 3x + 1)$	$-10pq(3pq + 4p - 5q^2)$	$5w(w^2 -7w + 3) -2w(2w^2 -5w + 2)$
$(x + 4)(x - 7)$	$(5x - 2y)(3x + 9y)$	$(z + 5)(4z - 6)$

Factoring Polynomials $(ax^2 + bx + c)$

Examples:

Factoring out GCF	Difference of squares	Perfect Square Trinomials	Trinomials
$6x^2 + 21x$	$x^2 - 64$	$4x^2 + 12x + 9$	$3x^2 + 7x + 2$
$3x(2x + 7)$	$(x + 8)(x - 8)$	$(2x + 3)^2$	$(3x + 1)(x + 2)$

Factor completely.

$6e^3f - 11ef$	$y^2 - 5y - 84$	$6x^2 + 7x + 2$
$6z^2 - 5z - 4$	$75x^2 - 147y^2$	$x^2 - 25$
$x^2 - 6x + 9$	$16c^2 + 72cd + 81d^2$	$x^4 - 16$

Solving Quadratics

Example $\qquad$ $x^2 + 3x^2 = 10$

$\qquad\qquad\quad$ $x^2 + 3x^2 - 10 = 0$ $\qquad\qquad$ *Subtract 10 from both sides so the quadratic is equal to 0*

$\qquad\qquad\quad$ $(x - 5)(x + 2) = 0$ $\qquad\qquad$ *Factor (see previous section on different kinds of factoring)*

$\qquad\qquad$ $x - 5 = 0 \quad$ and $x + 2 = 0$ $\qquad\qquad$ *Use Zero Product Property and put each binomial equal to 0*

$\qquad\qquad\quad$ $x = 5$ and $x = -2$ $\qquad\qquad$ *Solve both for x.*

Solve each quadratic to find the possible values of x.

$3x^2 - 12 = 0$	$6x^2 - 5x + 1 = 0$	$x^2 + 7x = 18$
$x^2 + 11x = 80$	$2x^2 = x + 15$	$3x^3 + 3x^2 - 60x = 0$

Distance Formula

Example: Find the distance between the points (-4,3) and (-7,8)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ *Distance formula*

$$d = \sqrt{(-7 - -4)^2 + (8 - 3)^2}$$ *Substitute the coordinates into the distance formula*

$$d = \sqrt{(-3)^2 + (5)^2}$$ *Simplify each parentheses*

$$d = \sqrt{9 + 25}$$ *Simplify by squaring each*

$$d = \sqrt{34}$$ *Combine like terms and make sure it is in simplest radical form.*

Find the distance between each pair of points

(3, 6) (0, -2)	(-3, 4) (1, 4)	(8, -2), (-3, -6)
(5, -2) (-6, 5)	(-6, -6) (-3, 2)	(-8, 5) (-1, 1)