Functions
Graphing a Variety of Functions
High School

Rationale

- To continue their mastery of functions, students must be able to graph a wide variety of functions, identifying and interpreting their various key features. This lesson allows students to practice with basic graphs of not only linear and quadratic functions but also square root, cube root, step, piecewise-defined, absolute value, polynomial, exponential, logarithmic, and trigonometric functions. By having a basic understanding of these myriad functions, students should be more comfortable when seeing even more complex functions in more advances mathematics, including calculus.

Goal

- To graph a variety of function types and point out their key features

Standards

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
- F-IF.7.a Graph linear and quadratic functions and show intercepts, maxima, and minima.
- F-IF.7.b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- F-IF.7.c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- F-IF.7.e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Objective

- Students will graph functions and study key features of their graphs.

Materials

- Activity sheet: ***Graphing Functions***
- Graphing paper/Coordinate graphs
- Optional: Graphing calculators or software

Procedures

- Begin a vocabulary discussion regarding the terms needed for this lesson. Initiate a conversation to get students to think about what the words probably mean if they do not already know, such as "MAX"ima, "MIN"ima, and "MID"line. First, have them think of the real-life contexts of the words/roots. Then, have them transfer their meanings to a mathematical context.
  - Vocabulary terms needed for this lesson include: intercept, maxima, minima, zeros of a function, end behavior, period, midline, and amplitude.
- Once students are familiar with the necessary vocabulary, divide them into groups of 2 or 3 and distribute the ***Graphing Functions*** worksheet to each student. Students may graph directly on the paper or use graphing paper, a graphing calculator, or graphing software.
- This lesson is a brief overview of the various types of functions. Sample functions are given for students to graph. Remind students to explain the vocabulary listed with their functions and to label/point out these aspects of their graphs.
- Once students have completed the worksheet, allow students to share their findings with other groups. Have them analyze their results, noting any differences in the visual representations across the groups' illustrations. Allow students opportunity for self-checking/correcting.
- Closing: Initiate a discussion to compare the attributes of all the different types of functions covered in the worksheet. Have students volunteer to compare and contrast the myriad properties of functions.

Teacher Tips

- Because mastering the concepts in this lesson requires so much vocabulary, have students create a vocabulary book/math dictionary or collage of the vocabulary involved in this lesson. Encourage them to be very visual in their examples. Students can then use their materials as a reference for this lesson and beyond.
- You may wish to have a brief review of the different types of functions that are going to be addressed in this lesson and have students talk about what is unique to each type and what the basic shapes of the graphs will look like.
- Some students may struggle with the usage of a calculator with such complex functions. You may need to present a brief tutorial on calculator usage, if only to demonstrate how students can access the lesser used operations on their calculators.
- Encourage students to label the location of the key vocabulary terms on their graphs.
- If some groups struggle with a particular function type and another group is able to graph it, have the successful group explain the process to the struggling group. Encourage camaraderie across the groups in the class.
- For easy reference, here is a list of vocabulary for this lesson:
  - **Amplitude** –the distance from the "resting" position (otherwise known as the mean value or average value) of the curve
  - **End behavior** – Even-degree polynomials are either up on both ends or down on both ends, depending on whether the polynomial has a positive or negative leading coefficient. Odd-degree polynomials have ends that go in opposite directions. If they

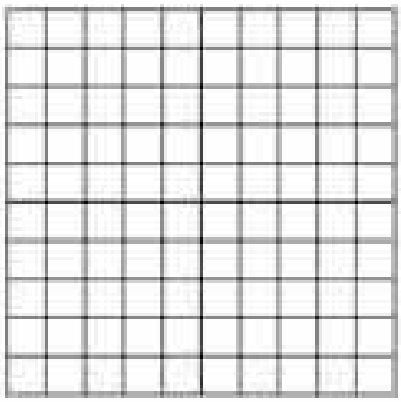
start down and go up, they are positive polynomials. If they start up and go down, they are negative polynomials. All even-degree polynomials behave like quadratics on their ends, and all odd-degree polynomials behave like cubics on their ends.

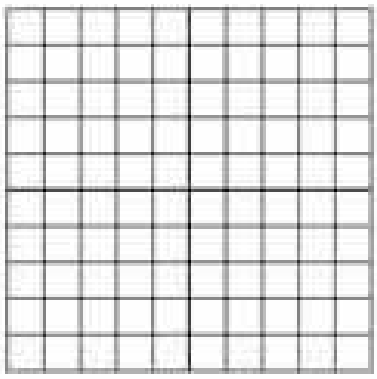
- **Maxima** – the largest value of a function
- **Minima** – the smallest value of a function
- **Midline** – a horizontal axis that is used as the reference line about which the graph of a periodic function oscillates
- **Period** –the distance (or time) that it takes for the sine or cosine curve to begin repeating again on a graph
- ***x*-intercept** – a point on the graph where y is zero
- ***y*-intercept** – a point on the graph where x is zero
- **Zeros (or roots) of a function** – the values where a function equals zero, found by setting a function each to zero, such as $f(x) = 0$, and solving

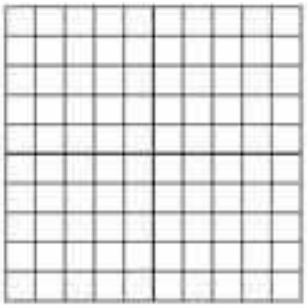
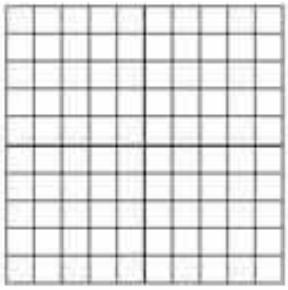
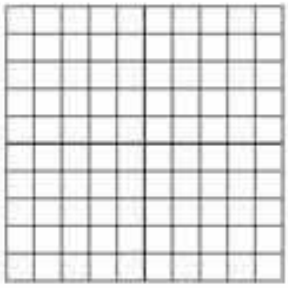
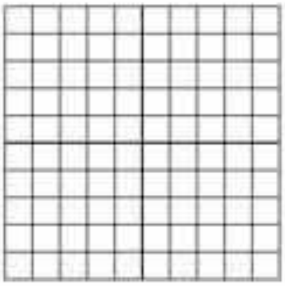
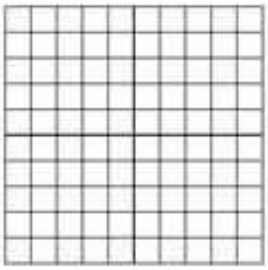
Extension Activities

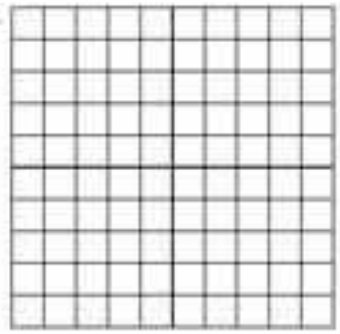
- Modify the complexity of the functions in this lesson depending on student abilities. For example, the absolute value function $y = |x|$ can be modified to $y = |x + 4|$ or $y = |x| + 4$ to further illustrate the effect of absolute value in a function.
- Encourage students to graph more complex functions, but make sure they have the use of technology if the difficulty level advances beyond the ability of students to graph the function by hand.
- Modify this lesson to be a jigsaw activity where each group will be responsible for learning one type of function very well. In a jigsaw activity, students start in a home group. The members are responsible for learning an assigned portion of a task assigned by the teacher. Then the teacher separates students into new groups, jigsaw groups, by assigning one member from each home group to a new group. If an activity begins with groups 1, 2, and 3, the jigsaw groups have a member from 1, 2, and 3. Students are then able to share with each other what they learned in their original group. This allows students to more thoroughly investigate particular types of functions and gives them a chance to teach about the attributes of a function type, thus providing a deeper understanding.

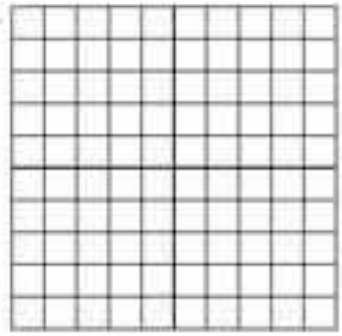
Graphing Functions

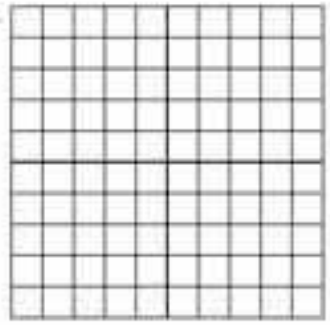
1. Linear Functions	Function	Graph
$y = mx + b$	$y = -2x$	
○ Intercepts		
○ Maxima		
○ Minima		

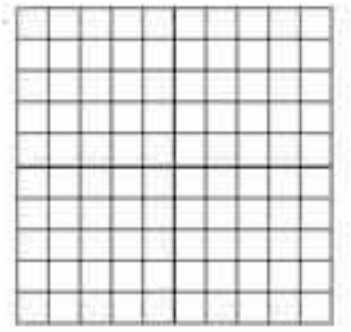
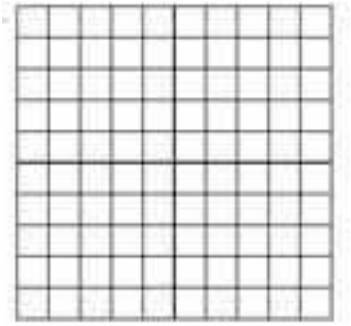
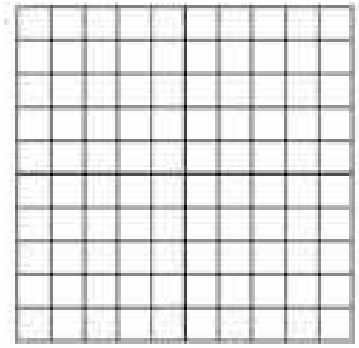
2. Quadratic Functions	Function	Graph
$y = ax^2 + bx + c$	$y = 2x^2$	
○ Intercepts		
○ Maxima		
○ Minima		

3.	Function	Graph
Square root function $f(x) = \sqrt{x}$	$y = \sqrt{(x-3)}$	
Cube root function $f(x) = \sqrt[3]{x}$	$f(x) = x^3$	
Piecewise-defined function $f(x) = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$	$f(x) = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$	
Step function $f(x) = \sum_{i=0}^{n} \alpha_i \chi_{A_i}(x)$	$f(x) = [[x - 1]]$	
Absolute value function $y = \lvert x \rvert$	$y = \lvert x \rvert$	

4. Polynomial functions	Function	Graph
$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$	$y = x^4 - 2x^2$	
○ Zeros		
○ End behavior		

5. Exponential functions	Function	Graph
$y = e^x$	$y = 2^x$	
○ Intercepts		
○ End behavior		

6. Logarithmic Functions	Function	Graph
$\log_b(xy) = \log_b(x) + \log_b(y)$	$y = \log_2 x$	
○ Intercepts		
○ End behavior		

7. Trigonometric Functions	Function	Graph
$y = \sin x$	$y = \sin 2x$	
$y = \cos x$	$y = \cos 3x$	
$y = \tan x$	$y = \tan 0.5x$	
○ Period		
○ Midline		
○ Amplitude		

Teacher copy with graphs included

Graphing Functions

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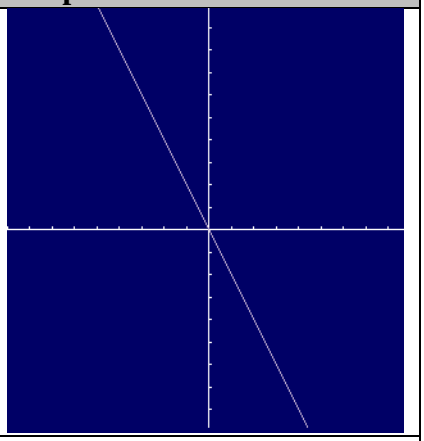
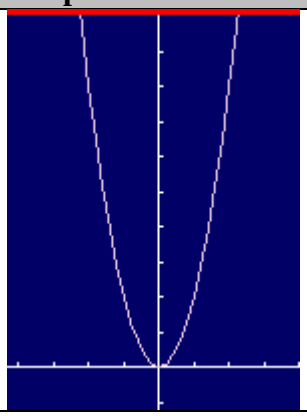
$y = mx + b$	$y = -2x$	
○ Intercepts		
○ Maxima		
○ Minima		

2. Quadratic Functions	Function	Graph
$y = ax^2 + bx + c$	$y = 2x^2$	
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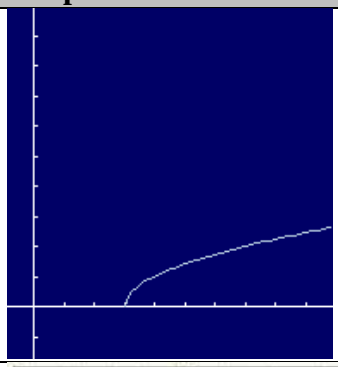
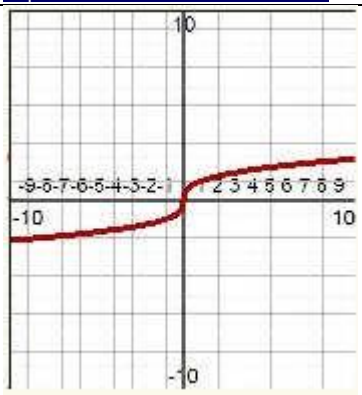
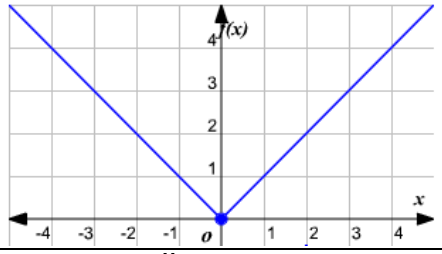
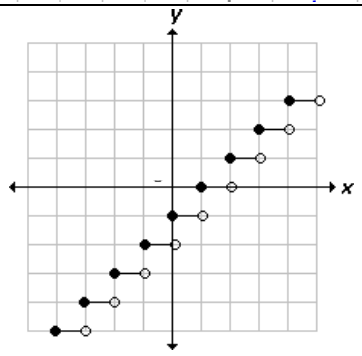
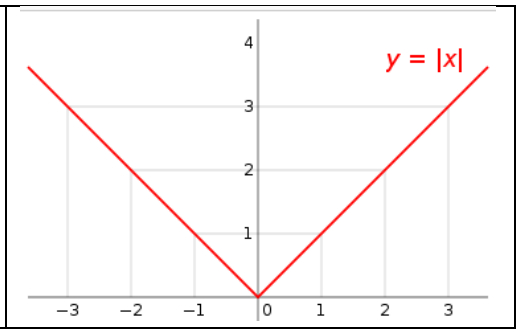
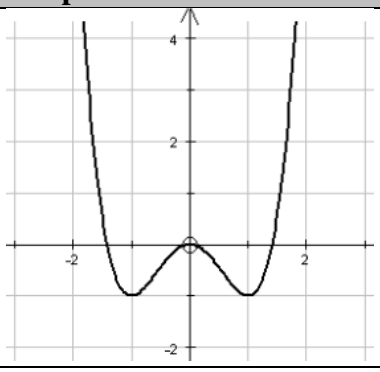
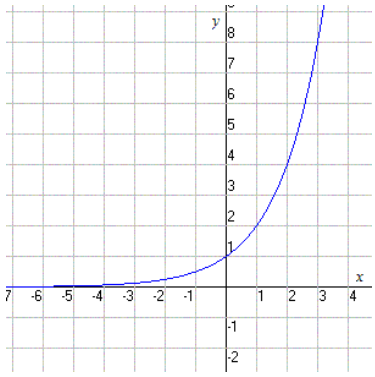
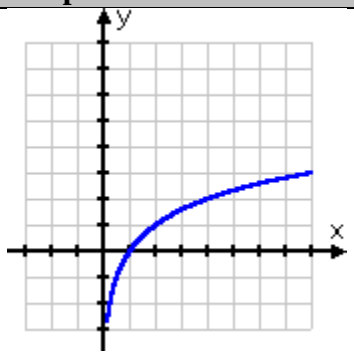
○ Maxima		
○ Minima		

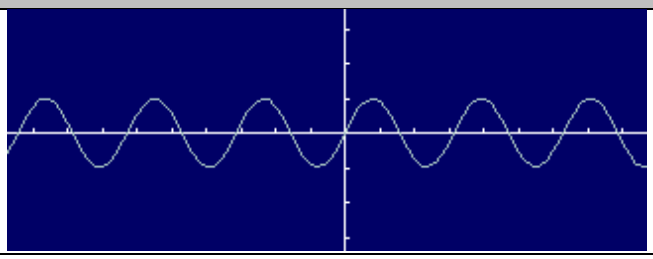
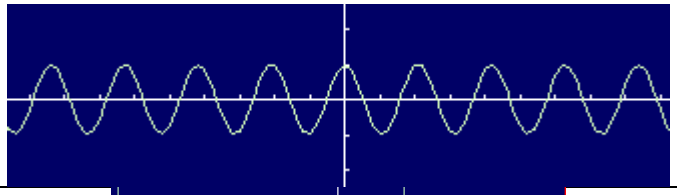
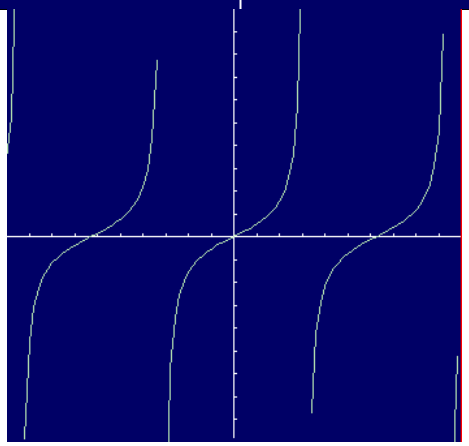
3.	Function	Graph
Square root function $f(x) = \sqrt{x}$	$y = \sqrt{(x-3)}$	
Cube root function $f(x) = \sqrt[3]{x}$	$f(x) = x^3$	
Piecewise-defined function $f(x) = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$	$f(x) = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$	
Step function $f(x) = \sum_{i=0}^{n} \alpha_i \chi_{A_i}(x)$	$f(x) = [[x - 1]]$	

Absolute value function $y = \|x\|$	$y = \|x\|$	

4. Polynomial functions	**Function**	**Graph**
$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x + a_0$	$y = x^4 - 2x^2$	
○ Zeros		
○ End behavior		

5. Exponential functions	**Function**	**Graph**
$y = e^x$	$y = 2^x$	
○ Intercepts		
○ End behavior		

6. Logarithmic Functions	Function	Graph
$\log_b(xy) = \log_b(x) + \log_b(y)$	$y = \log_2 x$	
○ Intercepts		
○ End behavior		

7. Trigonometric Functions	Function	Graph
$y = \sin x$	$y = \sin 2x$	
$y = \cos x$	$y = \cos 3x$	
$y = \tan x$	$y = \tan 0.5x$	
○ Period		
○ Midline		
○ Amplitude		