Lesson 2: Translations

Student Outcomes

- Students perform translations of figures on and off the coordinate plane.
- Students explore the properties of translations.
- Perform translations along given vectors.

Materials

- Personal white board, dry erase marker, and eraser (1 each per student)
- Rulers or straightedges and protractors should be available
- Pattern block (1 per student)
- Paper/Pencil (1 per student)
- Overhead transparency (1 per student)
- Whistle image (1 per student)
- Never-Sometimes-Always chart (1 per student)

Lesson Notes

This lesson introduces the concepts of basic rigid motions using informal language and prior knowledge of length and angle measure. Students experiment with rigid motions of two-dimensional figures on and off the coordinate plane. This lesson specifically focuses on translations. Students use a Never-Sometimes-Always graphic organizer to make sense of the properties of transformations they observe throughout the lesson. As they share their solutions, they understand that when figures are moved by a translation, length and angle measures are preserved. During the lesson debrief, encourage students to connect the idea that a translation is a type of rigid motion because length and angle measures are preserved.

Classwork

Exploratory Challenge: What happened to the figure? (7 minutes)

Direct students to take out their personal white boards and dry erase markers. Distribute one pattern block to each student (the shape does not matter). Direct students to place their blocks somewhere on their white boards.

- Take a moment to study your pattern block. What do you notice about this shape?
- Describe it to a partner;
  - My pattern block is a green triangle.
It looks like a square with 4 right angles.
Mine is a regular hexagon with 3 sets of opposite parallel sides and 6 obtuse angles.

Trace the outline of your pattern block on your white board. The drawing that shows the outline of your shape can be called a **figure**. Describe one thing about your figure to a partner. Start your sentence with “My figure has . . .”

- My figure has 3 sides.
- My figure has 4 right angles.
- My figure has 3 pairs of parallel lines.

Without flipping or turning your pattern block, slide it in any direction (model a sliding motion) to a new place on your white board. Describe to a partner the way you moved your block.

- I slid my block to the right, and then I moved it up.
- I slid my block down and to the left about 5 inches.
- I slid my block from the center to the top of my white board.

Trace the outline of your pattern block in its new place on your white board. How does the first figure compare to the second one?

- They look the same but in a different location.

When you slid your block across your white board and traced it, you performed a **translation**. The translation slides, or **maps**, one figure onto another.

Model a translation having the outline of a pattern block drawn on a piece of paper and the same outline on a transparency. Place them so the figures coincide, and then slide the transparency to a new location showing both images. Then, use the Never–Sometimes–Always chart to show how students’ understanding advances throughout this lesson. Take a quick poll of student thinking. At this point in the lesson, students may not be in total agreement. As students examine more translations, their thinking will solidify.

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Building Understanding Through Experience with Language

**Math Specification 1b**
In the previous lesson, students had to learn the definition of translations before practicing application of it. Now, students learn about the language of translations as they are actively building their understanding of that concept.

**Never Sometimes Always**

**Math Guideline 14**
This chart creates opportunities for teachers and students to demonstrate growth and changes in thinking by publicly recording and displaying examples of language students use to justify these statements.
**Exploratory Challenge: What does a translation need? (7 minutes)**

This activity establishes the need for a vector when translating. It utilizes a transparency to slide the original to its image location.

Pair students so they are sitting back to back. Give each student a transparency, a dry erase marker, eraser, and the whistle image from their student workbooks. Give the following directions to students and allow time for students to discuss their results. The goal is for students to determine that accuracy depends on direction and distance, which is what vectors provide.

- Partner A and Partner B should both put the paper copy of the whistle image under their own transparencies and trace their whistles onto the transparencies with dry erase markers.
- Partner A only should translate (slide) the transparency to a new place and carefully lifts the transparency slightly to draw the whistle image at its new place on the paper containing her original whistle image. Partner A should describe the location of the new image to Partner B so he can replicate the translation to the same place as Partner A's new whistle image.
- Partner B should only be listening to the directions and drawing, not asking questions, and not looking at Partner A's translation.
- When Partner B thinks he has drawn his second whistle image in the correct place, both partners stand up and compare the locations of their translated images.
- Was Partner B correct? Partner B should then suggest to Partner A how she could improve the instructions so that both drawings look the same.

**Support for Diverse Learners:**

Print the whistle on a sheet of grid paper. Students can use the grid lines to more precisely describe the translation.

You may need to allow students time to practice tracing an image from the transparency back onto their paper before starting the partner work.

Students who speak the same language could work together and discuss in their native language. Then, they can share with another group, in English, how they could make their directions more precise.

Consider adding to your word wall during this lesson. Organize synonyms together. For example, put the words *shape*, *figure*, *original*, and *image* together.

Repeat the activity with Partner B giving the directions. Debrief the activity with the class and introduce the term *image*.

- The original whistle on your paper is a figure, like the pattern block shapes you drew. When you performed a translation and traced the outline back onto your paper, you made a second figure. The second figure is called the *image* of the original figure.
• Describe the results of Partner A's directions.
  • Our images were basically in the place, but they were not exact.
  • She said to move the image over, and I wasn’t sure what that meant.
  • He told me to slide to the right, but I didn’t know how far to go.

• Describe the results of Partner B’s directions.
  • Our images ended up closer to the same spot than the first.
  • Our images were still a bit off, but I knew to move it up and left.
  • My partner told me to move 5 inches left, but I wasn’t sure how long 5 inches is without a ruler.

• Did Partner B’s directions change based on what was learned from Partner A? Say why.
  • Yes, I said to “go over,” but my partner knew not to say that and told me to move right, which was more specific.
  • Yes, he told me an actual distance of 5 inches to slide my image.
  • No, my partner still said to “go over,” and I didn’t know what that meant so I guessed.

• What parts of the directions were important to ensure the images matched?
  • It was important to say how far we slid the transparency.
  • It was important to give directions like “left” and “right” and not just say “over.”

Exploratory Challenge: Translation activity “Take 2” (8 minutes)

This activity establishes the need to define a vector and use it to describe a translation. The activity utilizes a straightedge, such as a ruler, a transparency, a piece of unlined paper, and a dry erase marker. Students also need the whistle image from the previous challenge.

Pair students (they do not need to be back to back for this exploration). Give Partner A a straightedge. One partner will draw the vector, and the other partner will perform the translation.

• It can be difficult to give clear directions to perform a translation. It is important to be precise about how far and in what direction to slide the transparency. Partner A should use a straightedge to draw an arrow on her paper. (Model drawing an arrow. Do not make it horizontal or vertical.) Your arrow does not need to be the same length or direction as mine. This arrow shows a length and a direction. We will use this arrow to perform a translation. Pass your paper to Partner B.

• Partner B should trace both the whistle and the Partner A’s arrow onto a transparency. Start with the traced arrow on top of the original arrow. Slide the transparency along the arrow so that the end of the traced arrow is at the tip of the original one that your partner drew. (Consider modeling this if students are struggling.)

• Partner A should hold down the paper while Partner B carefully lifts the transparency slightly to draw the whistle image at its new place on the paper. Did the image end at the location that Partner A intended? Why do you think that is?
  • Yes, the image did land in the location I intended because the arrow showed my partner the direction and the distance I wanted the image to move.
• No, my partner turned the transparency as they were sliding.
• No, my partner was not accurate when they were tracing.

• The arrows allow us to be more precise with our translation. The arrows we drew are called vectors. A vector is a directed line segment. Every vector has a specific length and direction. The tip of the arrow points to show the direction.

• With your partner, compare and contrast the vectors shown. What did you notice? (Chart student responses.)

![Vectors](image)

• I noticed that one has an arrow at B, and the other has an arrow at A.
• I noticed that they look like they are the same length.
• One says to go up and right; the other says to go down and left.

• These vectors are different because they go in opposite directions. How could we name these vectors using math symbols? Discuss with your partner.
  • We could call them both vector AB but put an arrow on top to show which direction they go. The first would be \( \overrightarrow{AB} \) and the second would be \( \overrightarrow{BA} \).
  • We could call one \( \overrightarrow{AB} \) and the other \( \overrightarrow{BA} \).

(Label the vectors with the names)

**Support for Diverse Learners:**
Create an anchor chart of the word vector and directed line segment on the chart. Continue to add to the chart as you draw and label the vectors during the upcoming discussion. Consider making two vectors to form the “V” in the word vector to provide a visual with the word.

Ask students to revisit the Never–Sometimes–Always graphic organizer to update their thinking. Circulate to listen in on conversations and make note of students who could share with the class.

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Exercises 1-2 (12 minutes)

Exercise 1

Provide time for students to practice translating along a vector. Students should make three copies of the original image, S. As you circulate, monitor to make sure students are returning to the original image each time they perform a translation.

- Which vector did you use to translate first?
  - Answers will vary.

- We need a way to tell the difference between the original figure and its images. To label our images we use a small tick mark. We use the same letter as the original figure to indicate the image is related to the original figure and small tick marks to tell them apart. When we make this tick mark, we say “prime.” If we use more than one tick mark, we can say “double prime” for two tick marks, and “triple prime” for three tick marks.

- Label the image you created first with $S'$. Now say “S prime” and point to the image you labeled. Label the image you created second with $S''$. Now say “S double prime” and point to the image you labeled. Finally, label the last image you created with $S'''$. Now say “S triple prime” and point to the image you labeled.

- **Prime**
- **Double prime**
- **Triple prime**

Use a transparency and the vectors given to create three images of figure S.

- Which vector was used to translate figure S to $S'''$?
  - Answers may vary. For example, Vector EF was used to translate figure S to $S'''$. 
Exercise 2

Provide students time to work independently or in small groups on this exercise. Encourage them to use tools such as transparencies, rulers, and protractors to support their answers. Select some students to share their work with the entire class.

- A translation by vector $HI$ is shown on the coordinate plane. Look at each figure on the coordinate plane and write in the missing labels. Then, precisely describe how the segment, the line, and the angle were translated. (Provide students time to work and discuss their ideas.)

- What happened to the length of segment $AB$ when it was translated to segment $A'B'$? How do you know?
  - The length stayed the same. I measured with my ruler. I used my transparency to help me.

- What happened to the measure of angle $CDE$ when it was translated to angle $C'D'E'$? How do you know?
  - The angle measure stayed the same. I measured it with my protractor. I used my transparency to trace the original then slid it on top of angle $C'D'E'$.

- What happened to line $FG$ when it was translated to line $F'G'$? How do you know?
  - The line is parallel to the original line because there was no turning it. Since lines go on forever in both directions, it will be parallel to the original no matter how I translate it.
Closing (5 minutes)

As you close this lesson, revisit the Never–Sometimes–Always graphic organizer one final time. Ask students who changed their minds to share their thinking. Some students might still be unclear about parallel lines moving to parallel lines. These ideas will be solidified as students continue to study rigid motions in later lessons.

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Exit Ticket (5 minutes) - student worksheet below
Lesson 2: Translations

Exit Ticket

Use the figures on the coordinate plane to answer the questions.

1. Explain the motions that would place segment $AB$ onto segment $A'B'$. Use the words right, left, up, down, and the number of units in your explanation.

2. Name the vector that was used to translate segment $AB$ to $A'B'$.

3. If the length of segment $AB$ is 3.6 units, what is the length of segment $A'B'$? Why?

4. Translate segment $AB$ by vector $EF$. Draw the image after the translation using prime notation to label the image.
Lesson 2: Translations

Exit Ticket Sample Solutions

Use the figures on the coordinate plane to answer the questions.

1. Explain the motions that would place segment $AB$ onto segment $A'B'$. Use the words right, left, up, down, and the number of units in your explanation.

   Segment $AB$ would translate down 3 units and right 1 unit.

2. Name the vector that was used to translate segment $AB$ to $A'B'$.

   Vector $CD$

3. If the length of segment $AB$ is 3.6 units, what is the length of segment $A'B'$? Why?

   The length of segment $A'B'$ is 3.6 units because translations are rigid motions, which preserve length.

4. Translate segment $AB$ by vector $EF$. Draw the image after the translation using prime notation to label the image.

   Verify that students have translated segment $AB$ up 3 units and left 1 unit and labeled using double primes as shown below.