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Understanding Volatility

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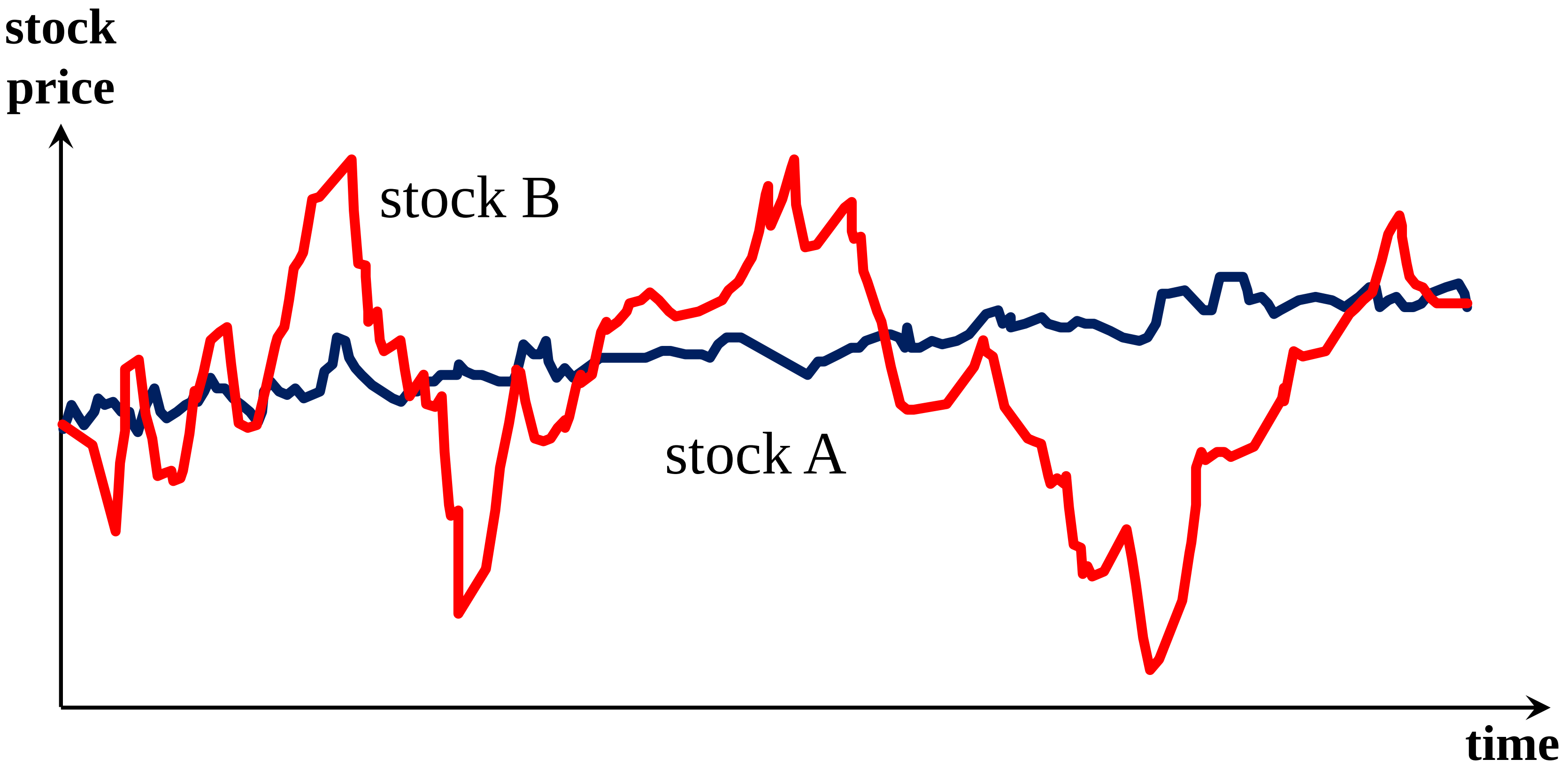
What is volatility?

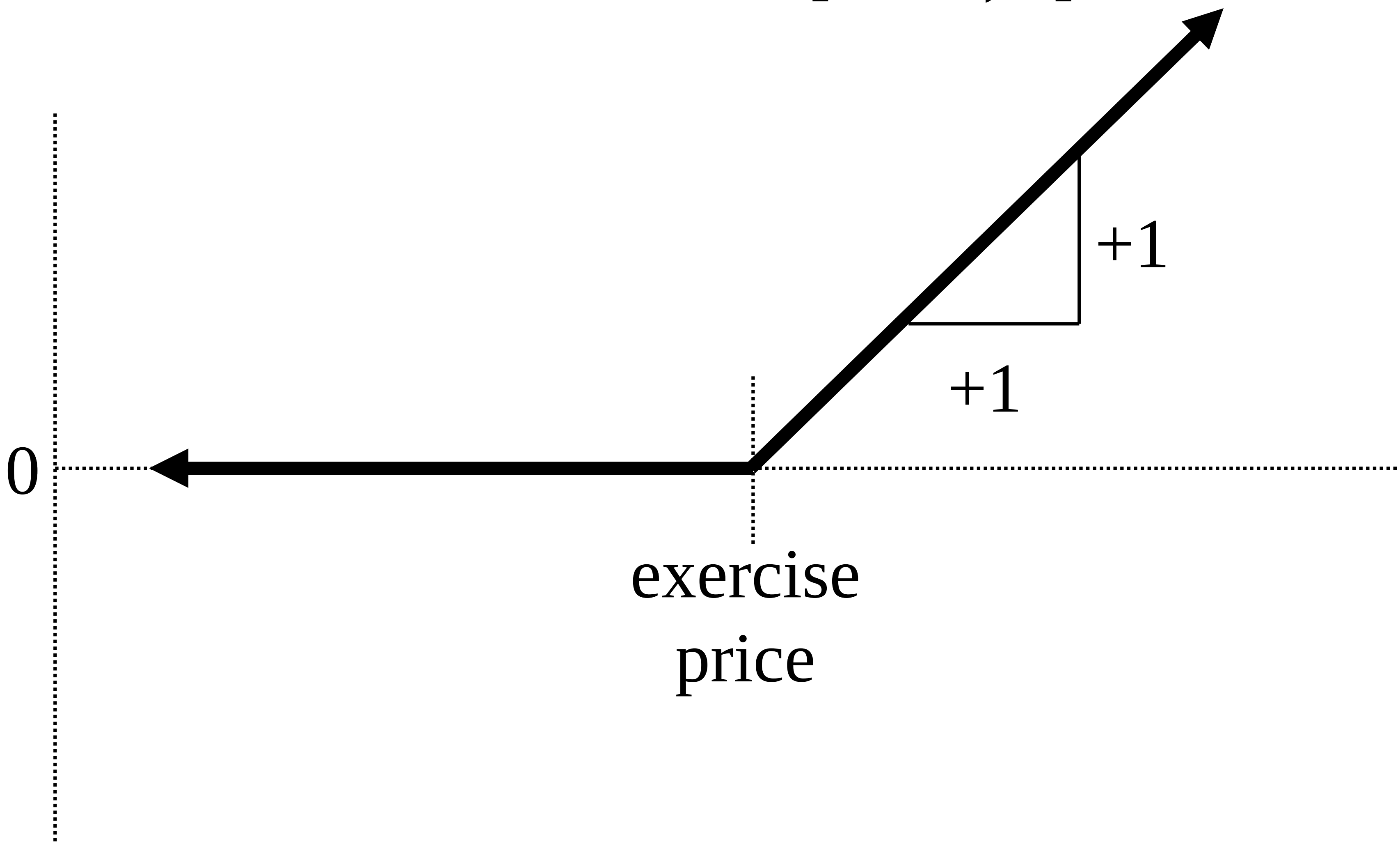


Which stock is more volatile?

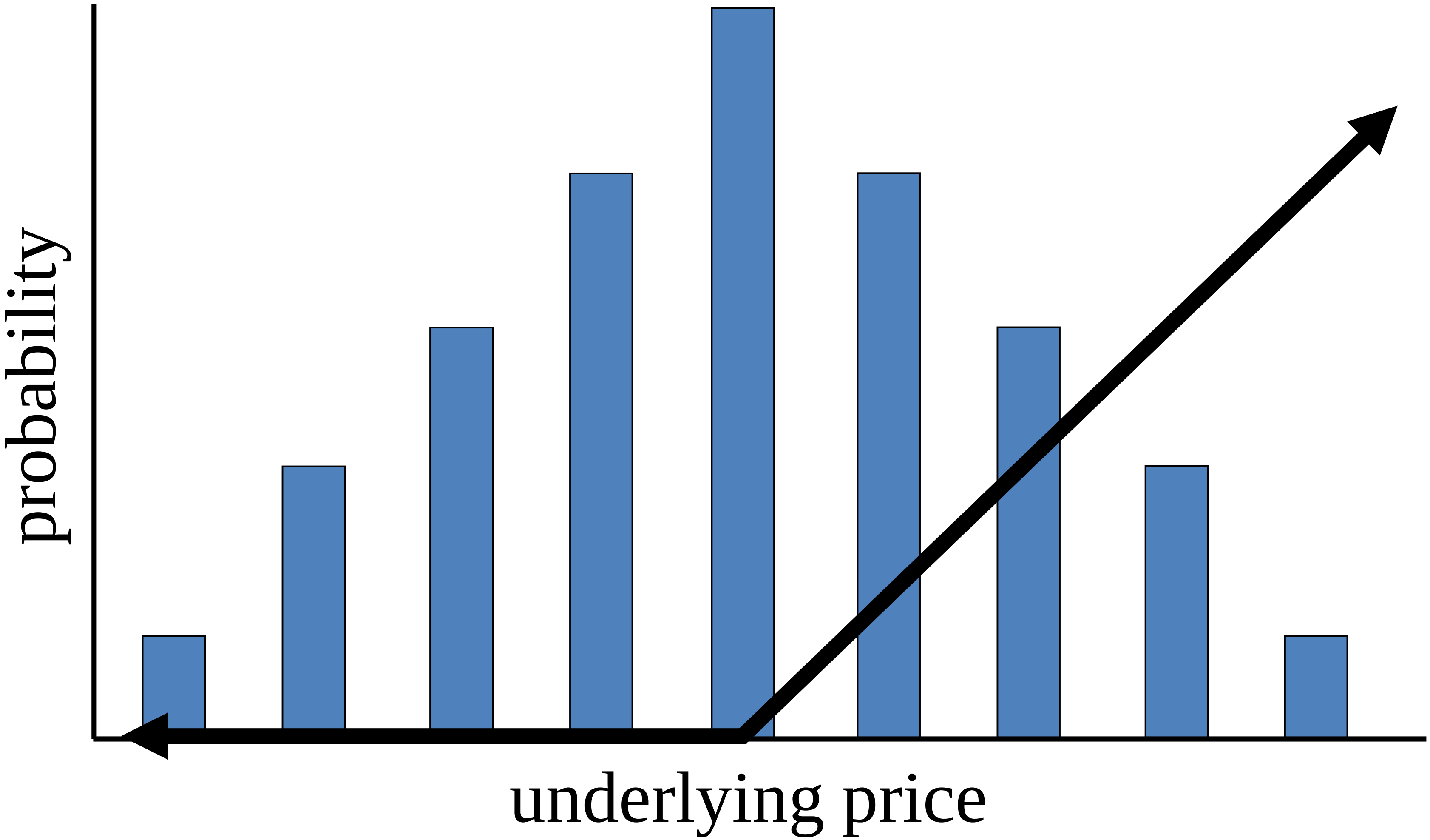
Volatility is a measure of ***how*** we arrive, rather than ***where*** we arrive.

What is the value of a call option at expiration?

intrinsic value: maximum [S - X, 0]



We might propose a probability distribution of underlying prices at expiration. To evaluate an option we can overlay the option's intrinsic value on our probability distribution.



For each underlying price, S_i, we have an intrinsic value and a probability, p.

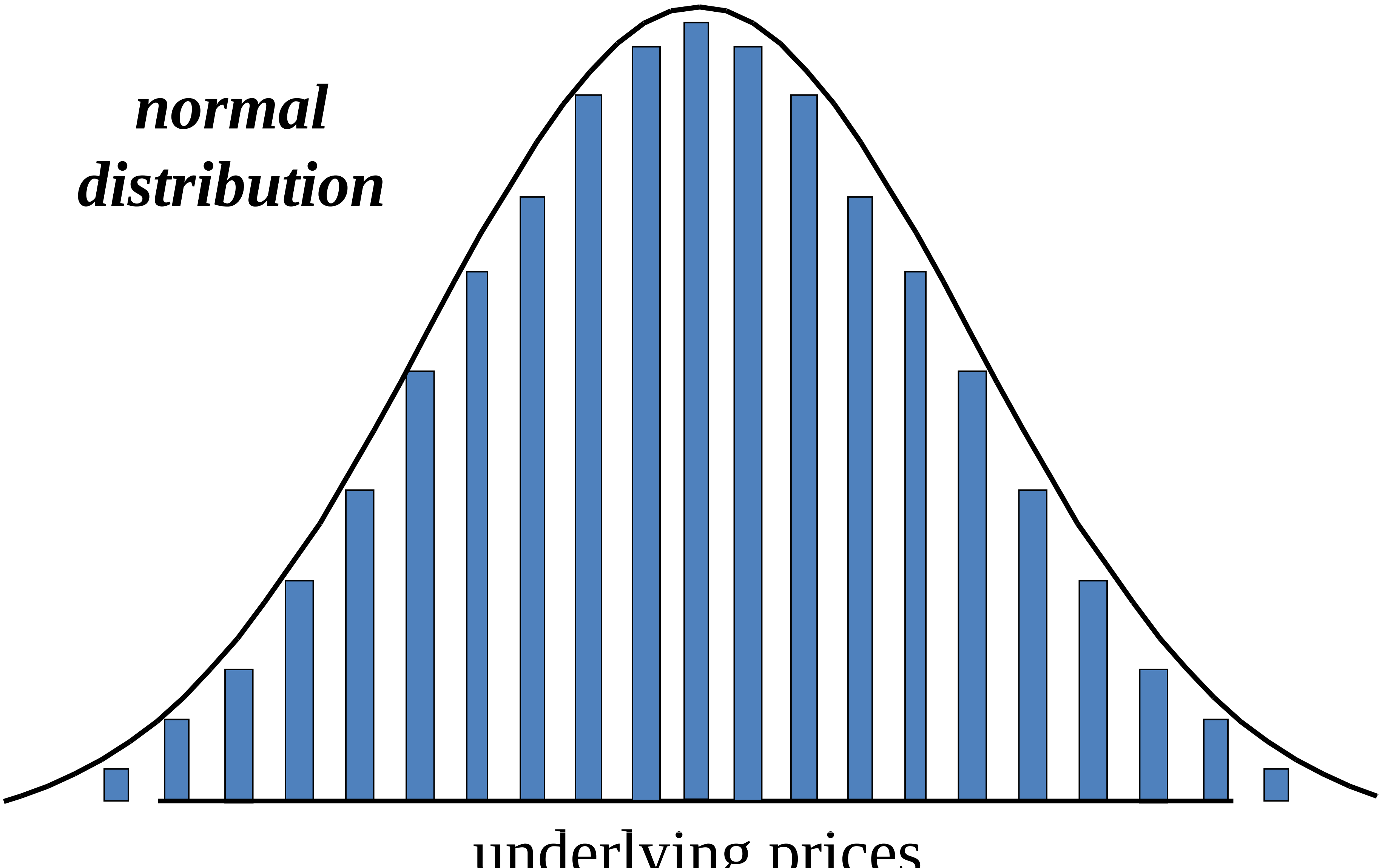
$$p * \text{intrinsic value} = p * \text{maximum}[S - X, 0]$$

The expected value for the option at expiration is the sum of all these individual values.

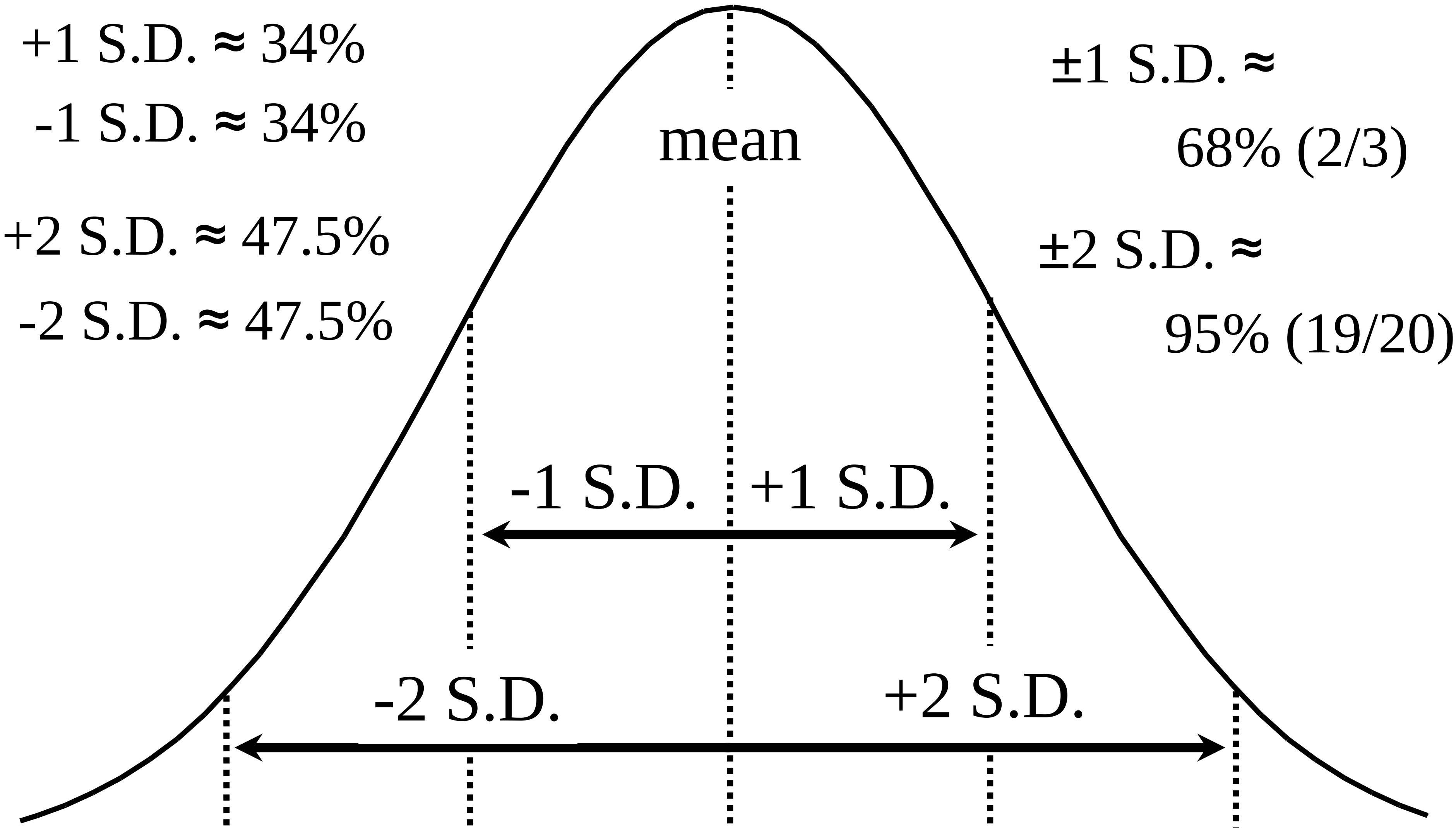
$$\sum_{i=1}^{n} p_i * \text{maximum}[S_i - X, 0]$$

The theoretical value is the present value of this amount.

What probability distribution should we assume for the underlying contract?



All normal distributions are defined by their mean (μ) and standard deviation (σ).



exercise price

time to expiration

underlying price

interest rate

volatility

(dividends)

mean?

standard deviation?

Mean – forward price

(underlying price, time to expiration, interest rates, dividends)

stock: $S * (1+r*t) - D$

foreign currency: $S * \dfrac{1+r_d*t}{1+r_f*t}$

futures contract: F

Standard deviation – volatility

Volatility: one standard deviation, in percent, over a one year period.

1-year forward price = 100.00

volatility = 20%

One year from now:

- 2/3 chance the contract will be between 80 and 120 (100 ± 20%)

- 19/20 chance the contract will be between 60 to 140 (100 ± 2*20%)

- 1/20 chance the contract will be less than 60 or more than 140

1-year later

underlying price = 180

Was 20% an accurate volatility?

If 20% was correct, how many standard deviations did the market move? (180-100) / 20 = 4

What is the likelihood of a 4 standard deviation occurrence? $\approx$ 1 / 16,000

Is one chance in 16,000 impossible?

What does an annual volatility tell us about movement over some other time period?

monthly price movement?

weekly price movement?

daily price movement?

$$\text{Volatility}_t = \text{Volatility}_{\text{annual}} * \sqrt{t}$$

Daily volatility (standard deviation)

Trading days in a year? 250 – 260

Assume 256 trading days

$t = 1/256$ $\sqrt{t} = \sqrt{1/256} = 1/16$

$$\text{Volatility}_{\text{daily}} = \text{Volatility}_{\text{annual}} / 16$$

current price = 100.00

$\text{volatility}_{\text{daily}} \approx 20\% / \boldsymbol{16} = 1\tfrac{1}{4}\%$

One trading day from now:

- ***2/3*** chance the contract will be trading between 98.75 and 101.25

  (100 ± 1¼%)

- ***19/20*** chance the contract will be trading between 97.50 and 102.50

  (100 ± 2*1¼%)

Weekly volatility:

$$t = 1/52 \qquad \sqrt{t} = \sqrt{1/52} \approx 1/7.2$$

$$\text{Volatility}_{\text{weekly}} \approx \text{Volatility}_{\text{annual}} / 7.2$$

Monthly volatility:

$$t = 1/12 \qquad \sqrt{t} = \sqrt{1/12} \approx 1/3.5$$

$$\text{Volatility}_{\text{monthly}} \approx \text{Volatility}_{\text{annual}} / 3.5$$

stock = 64.75; volatility = 31.0%

daily standard deviation?

$\approx$ 64.75 * 31% / 16

= 64.75 * 1.94% $\approx$ **1.25**

weekly standard deviation?

$\approx$ 64.75 * 31% / 7.2

= 64.75 * 4.31% $\approx$ **2.79**

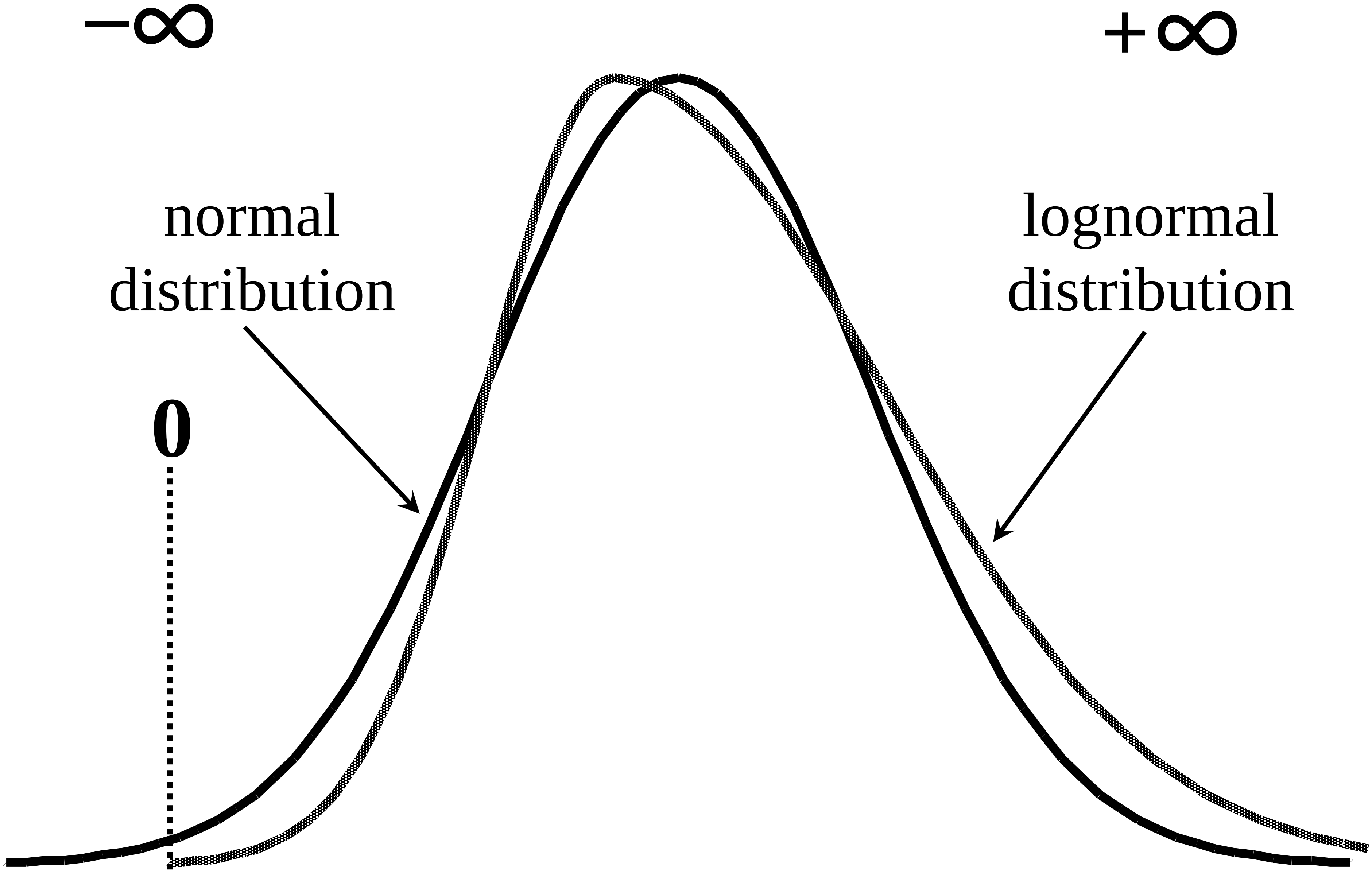
stock = 64.75; volatility = 31.0%

daily standard deviation $\approx$ 1.25

+.50 +.95 -.70 -1.15 +.65

Is 31% a reasonable volatility estimate?

How often do you expect to see an occurrence greater than one standard deviation?



−∞
+∞
normal distribution
lognormal distribution
0

forward price = 100

	normal distribution	lognormal distribution	price
110 call	3.00	3.20	2.90
90 put	3.00	2.80	3.10

Are the options mispriced?

Maybe the marketplace thinks the model is wrong.

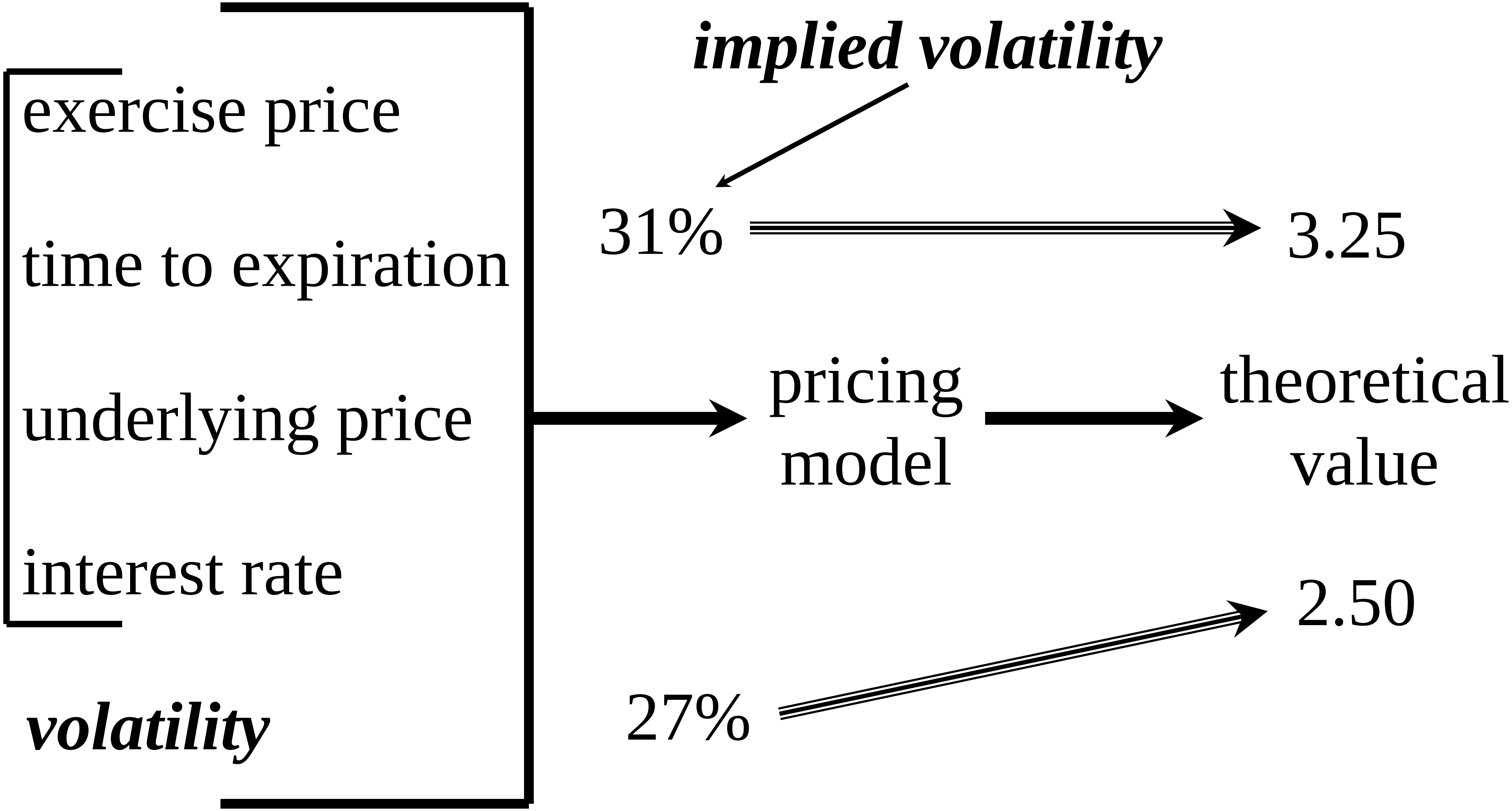
Maybe the marketplace is right.

Option traders interpret volatility data in a variety of ways. The two most common interpretations are….

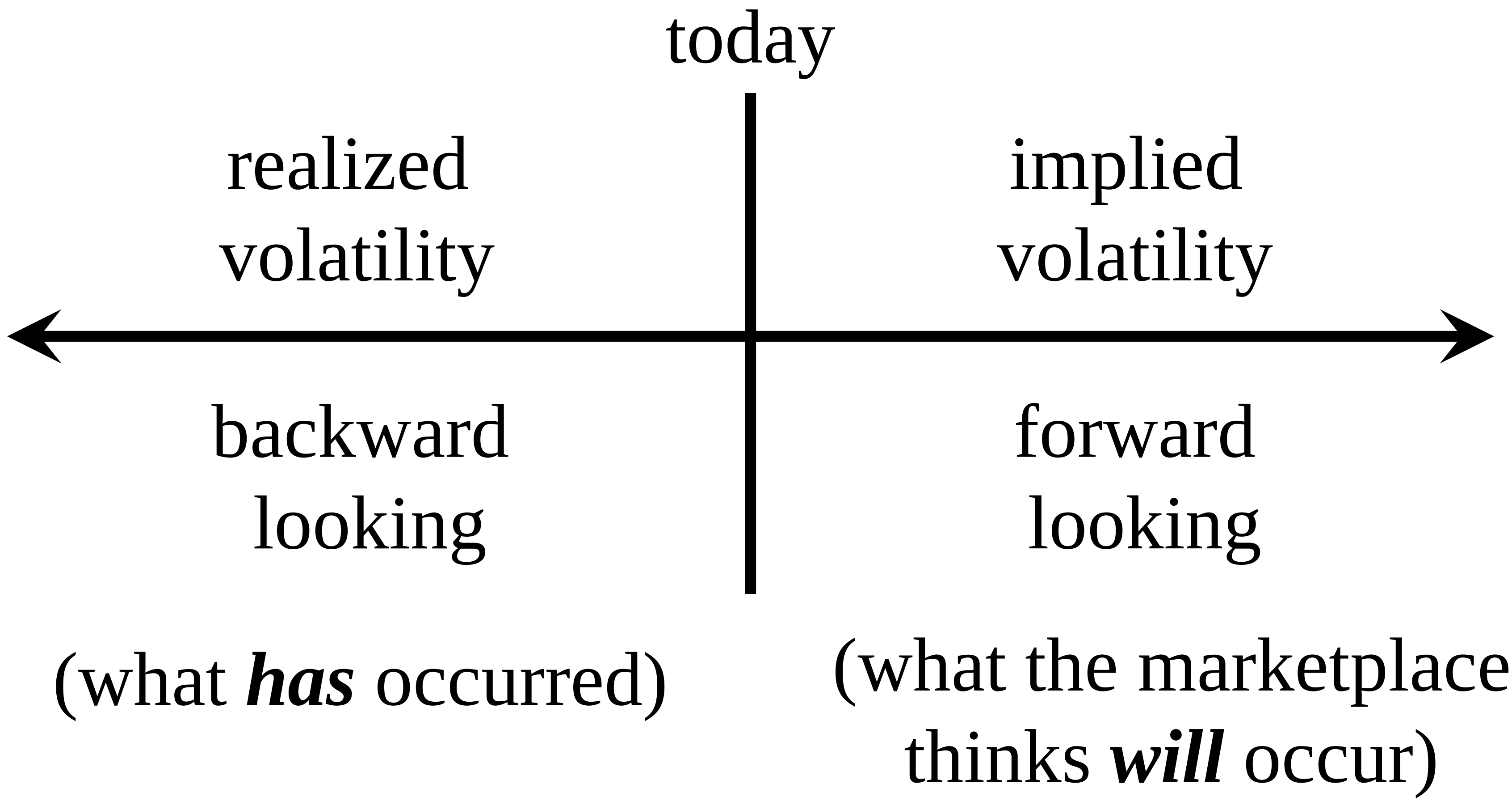
realized volatility: The volatility of the underlying contract over some period of time.

implied volatility: The marketplace's consensus forecast of future realized volatility as derived from option prices in the marketplace.

Vega – the sensitivity of an option's price to a change in implied volatility.

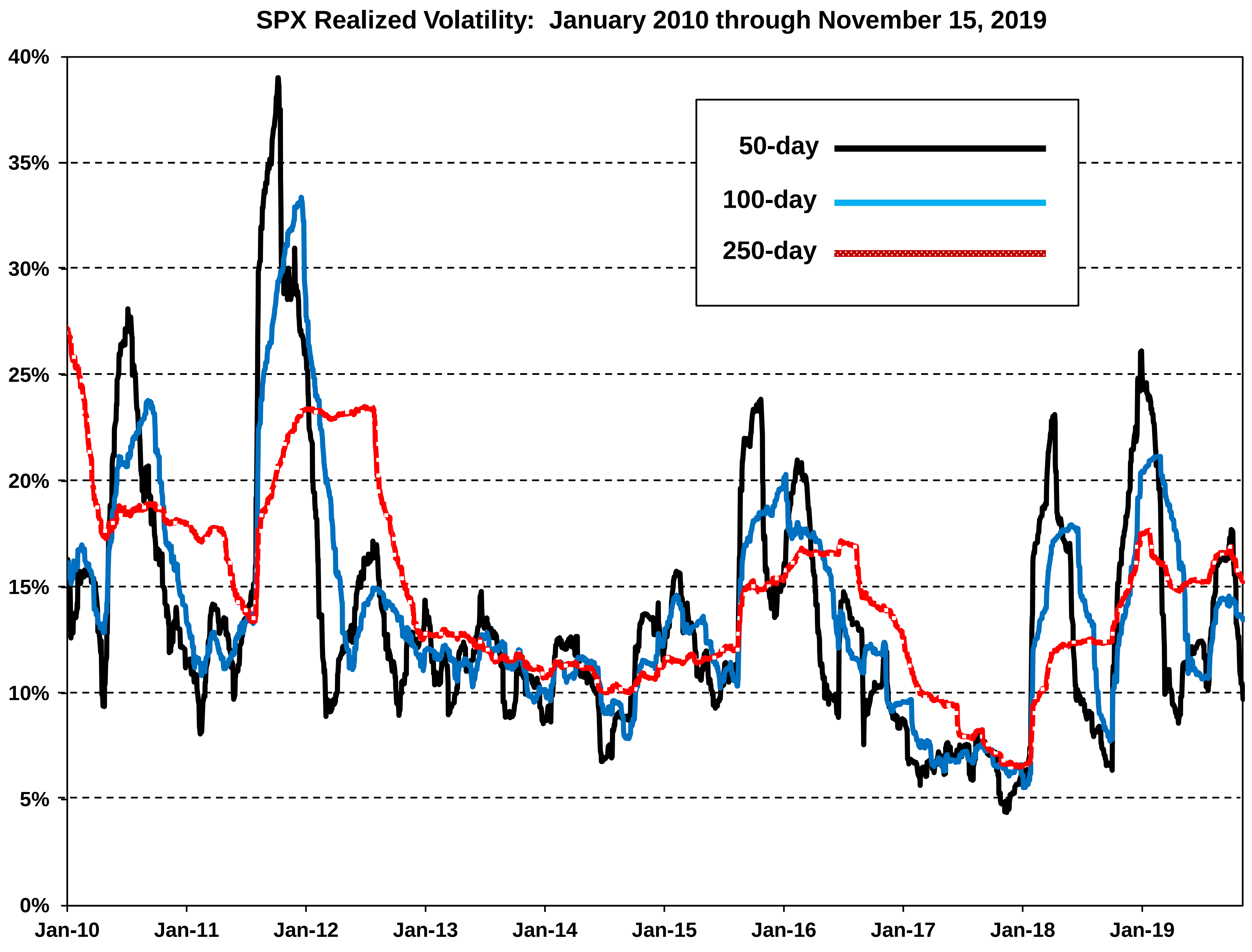


exercise price
time to expiration
underlying price
interest rate
volatility
implied volatility
31%
3.25
pricing
model
theoretical
value
2.50
27%



implied volatility = price

realized volatility = value



SPX Realized Volatility: January 2010 through November 15, 2019
40%
35%
30%
25%
20%
15%
10%
5%
0%
Jan-10
Jan-11
Jan-12
Jan-13
Jan-14
Jan-15
Jan-16
Jan-17
Jan-18
Jan-19
50-day
100-day
250-day

November 15, 2019 **Time to January expiration = 9 weeks**

SPX = 3120.46 **January forward price = 3120.75**

Interest rate = 2.00%

January	price	theoretical value if volatility is…. 11%	13%	15%	implied volatlity
2925 call	214.60	200.02	204.30	209.65	16.61%
3125 call	55.85	54.67	64.98	75.29	11.23%
3325 call	2.60	5.53	10.26	16.13	9.29%
2925 put	19.20	4.86	9.14	14.49	16.54%
3125 put	60.10	58.90	69.22	79.53	11.23%
3325 put	205.85	209.17	213.90	219.76	9.00%

November 15, 2019 **Time to January expiration = 9 weeks**

SPX = 3120.46 **January forward price = 3120.75**

Interest rate = 2.00%

January		11% → 13%		increase	%
2925 call	ITM	200.02	204.30	4.28	2%
3125 call	ATM	54.67	64.98	10.31	19%
3325 call	OTM	5.53	10.26	4.73	86%

1. In total points an ***at-the-money*** option is always more sensitive to a change in volatility than an equivalent in- or out-of-the-money option.

2. In percent terms an ***out-of-the-money*** option is always more sensitive to a change in volatility than an equivalent in- or at-the-money option.

November 15, 2019 **Time to January expiration = 9 weeks**

SPX = 3120.46 **January forward price = 3120.75**

Interest rate = 2.00%

January		11% →	13%	increase	%
2925 call	ITM	200.02	204.30	4.28	2%
3125 call	ATM	54.67	64.98	10.31	19%
3325 call	OTM	5.53	10.26	4.73	86%
2925 put	OTM	4.86	9.14	4.28	88%
3125 put	ATM	58.90	69.22	10.32	18%
3325 put	ITM	209.17	213.90	4.73	2%

November 15, 2019

January expiration = 9 weeks **March expiration = 18 weeks**

January forward = 3120.75 **March forward = 3120.30**

January	call price	11% →	13%	increase	implied
2925 call	214.60	200.02	204.30	4.28	16.61%
3125 call	55.85	54.67	64.98	10.31	11.23%
3325 call	2.60	5.53	10.26	4.73	9.29%
March					
2925 call	243.05	210.29	219.51	9.22	17.37%
3125 call	94.05	77.70	92.24	14.54	13.25%
3325 call	14.05	17.79	27.77	9.98	10.16%

1. In total points an ***at-the-money*** option is always more sensitive to a change in volatility than an equivalent in- or out-of-the-money option.

2. In percent terms an ***out-of-the-money*** option is always more sensitive to a change in volatility than an equivalent in- or at-the-money option.

3. A ***long-term*** option is always more sensitive to a change in volatility than an equivalent short-term option.

Volatility Trading

Volatility trading has been a cornerstone of option trading since the CBOE first opened in 1973.

Traders have used option strategies to “buy” and “sell” volatility, attempting to profit from changes in implied volatility, or to capture differences between implied volatility and the realized volatility of the underlying contract.

A fundamental rule of volatility trading

If implied volatility is low, prefer strategies with a positive vega.

If implied volatility is high, prefer strategies with a negative vega.

High or low compared to what….?

high or low compared to the historical range of implied volatility, or high or low compared to the expected realized volatility of the underlying contract.

Volatility Trading

Common volatility strategies:

- straddles
- strangles
- butterflies
- ratio spreads
- calendar spreads

These strategies can be used to “buy” or “sell” volatility.

Volatility Trading

In addition to "pure" volatility trading strategies, volatility also has important, implications for other types of option strategies.

You are bullish on a stock which is currently trading at 70.00.

You are considering one of two 5-point bull call spreads, the 65 / 70 spread, and the 70 / 75 spread (buy the lower strike, sell the higher).

Volatility Trading

In addition to "pure" volatility trading strategies, volatility also has important, implications for other types of option strategies.

You are bullish on a stock which is currently trading at 70.00.

1. buy the 65 call / sell the 70 call
2. buy the 70 call / sell the 75 call

Are the spreads essentially the same? Might you prefer one spread over the other? Why?

Since an at-the-money option has a greater vega than an in-the-money or out-of-the-money option….

If implied volatility is low, prefer to buy the at-the-money option.

buy the 70 call / sell the 75 call

If implied volatility is high, prefer to sell the at-the-money option.

buy the 65 call / sell the 70 call

stock price = 70 time to expiration = 3 months

			implied volatility	
	value		low	high
Volatility	25%	Δ	20%	30%
65 call	6.43	73	5.90	7.01
65 / 70 spread	**2.95**	**21**	3.11	2.83
70 call	3.48	52	2.79	4.18
70 / 75 spread	**1.84**	**21**	1.74	1.89
75 call	1.64	31	1.05	2.29

Almost every trade involving options has a volatility component. Even trades which do not seem to be sensitive to volatility often have volatility implications

Consider one of the most common investment strategies involving options: the sale of a covered call, or ***buy/write***. In this strategy a call option is sold against a holding in an underlying stock.

At first glance a buy/write trade does not seem to be affected by volatility. But closer examination may show that this is not necessarily the case.

A portfolio manager (or investor) has two goals:

1. Increase the value of the portfolio

2. Outperform the market (or some benchmark)

Suppose a portfolio manager implements a covered call writing program against a portfolio of stocks. Under what conditions will the manager achieve both goals?

If the market makes a big upward move the manager will achieve goal number ***1***, but not goal number ***2*** since the short calls limit the upside profit potential.

A portfolio manager (or investor) has two goals:

1. Increase the value of the portfolio

2. Outperform the market (or some benchmark)

Suppose a portfolio manager implements a covered call writing program against a portfolio of stocks. Under what conditions will the manager achieve both goals?

If the market makes a big downward move the manager will achieve goal number ***2***, but not goal number ***1*** since the short calls offer only limited downside protection.

A portfolio manager (or investor) has two goals:

1. Increase the value of the portfolio

2. Outperform the market (or some benchmark)

The portfolio manager will only achieve both goals if the market is relatively quiet, making only a small upward or downward move.

A covered call writing program is therefore a ***short volatility*** position. It performs best in non-volatile markets.

Some Basic Volatility Characteristics

Volatility is similar to the weather

Today's high temperature is 20°

If you have no other information, what's your best guess about tomorrow's high temperature?

18° 20° 22°

Volatility is ***serial correlated***: in the absence of other information, the best guess about volatility over the next period of time is the volatility that occurred over the last period of time.

Some Basic Volatility Characteristics

Volatility is similar to the weather

Today's high temperature is 20°

The average high temperature at this time of year is 23°. Now, what's your best guess about tomorrow's high temperature?

18° 20° 22°

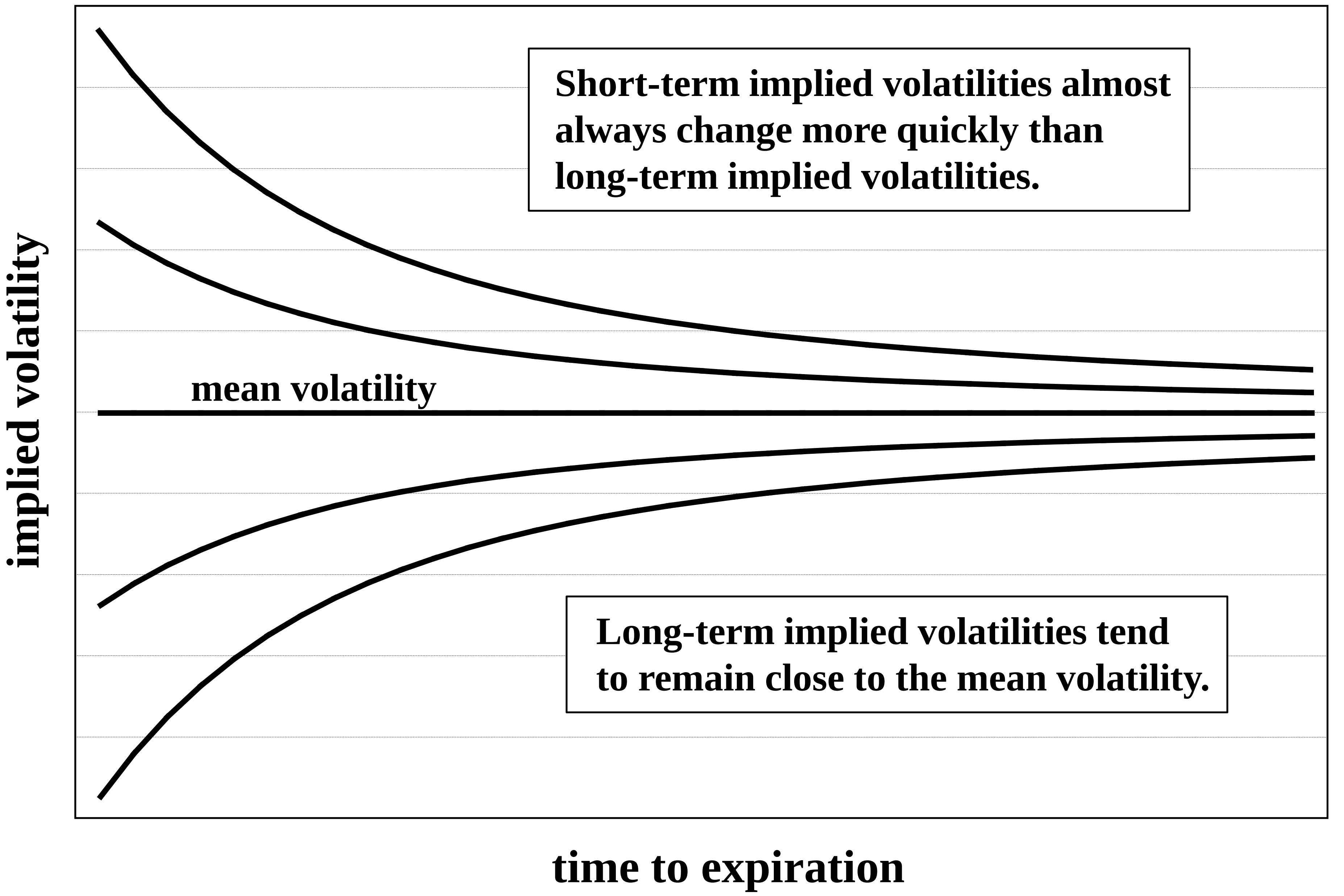
Volatility is ***mean reverting***: it tends to revert to its long-term average. This is especially true over long periods of time.

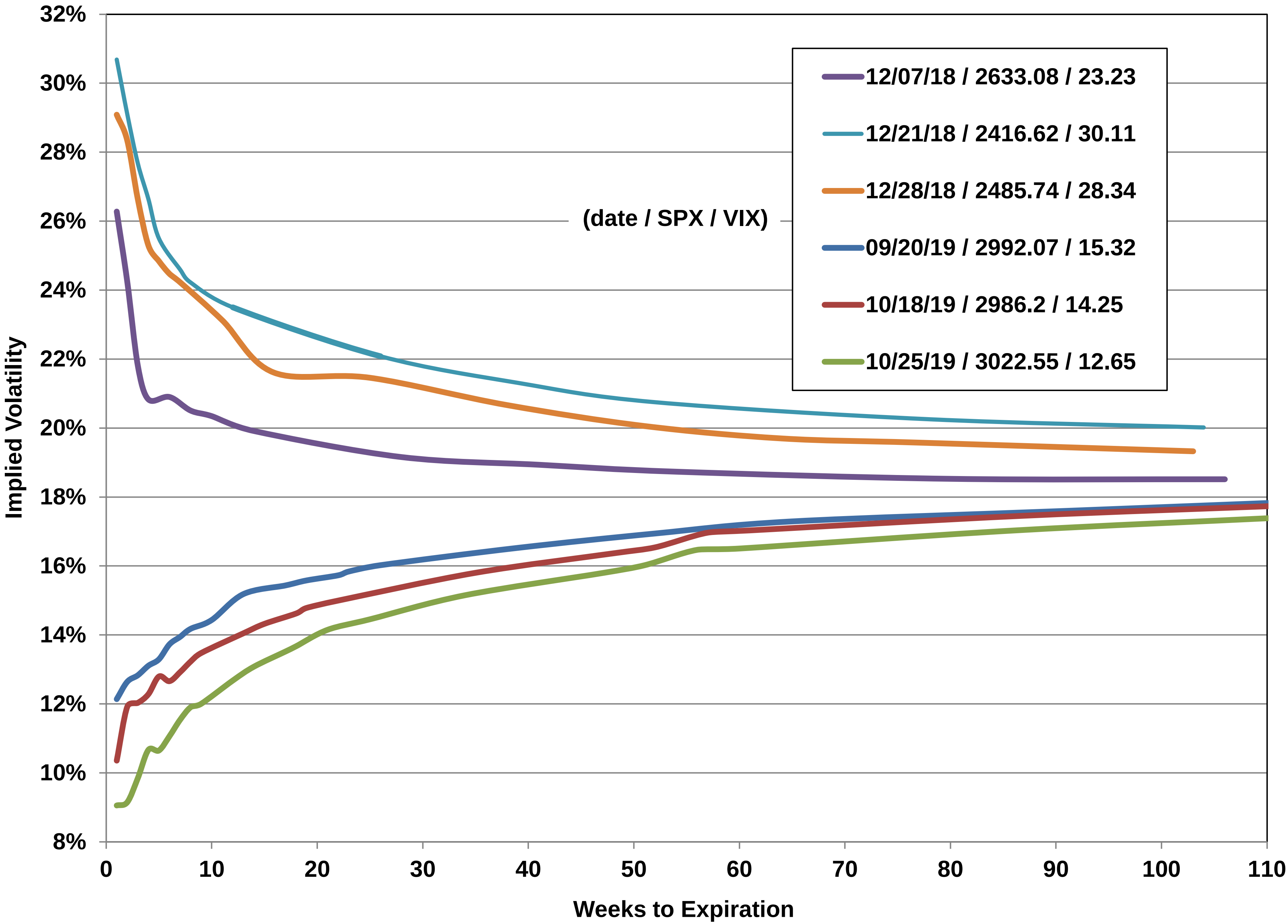
Some Basic Volatility Characteristics

Mean volatility = 20%

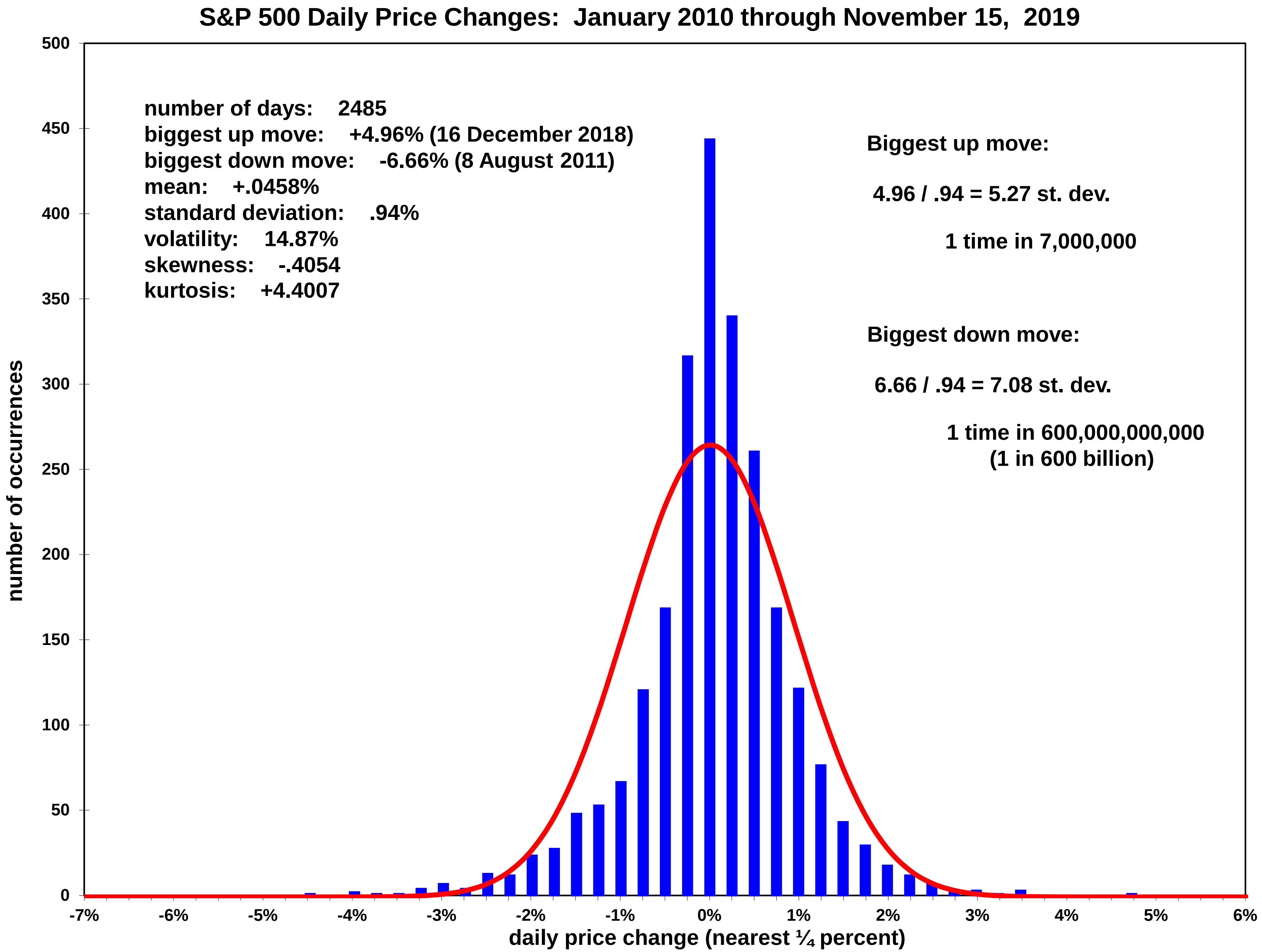
	current imp. vol.	if imp. vol. rises	if imp. vol. falls
March implied	20%	25%	15%
June implied	20%	23%	17%
September implied	20%	21%	19%

Term Structure of Volatility





32%
30%
28%
26%
24%
22%
20%
18%
16%
14%
12%
10%
8%
Implied Volatility
0
10
20
30
40
50
60
70
80
90
100
110
Weeks to Expiration
(date / SPX / VIX)
12/07/18 / 2633.08 / 23.23
12/21/18 / 2416.62 / 30.11
12/28/18 / 2485.74 / 28.34
09/20/19 / 2992.07 / 15.32
10/18/19 / 2986.2 / 14.25
10/25/19 / 3022.55 / 12.65



S&P 500 Daily Price Changes: January 2010 through November 15, 2019
number of days: 2485
biggest up move: +4.96% (16 December 2018)
biggest down move: -6.66% (8 August 2011)
mean: +.0458%
standard deviation: .94%
volatility: 14.87%
skewness: -.4054
kurtosis: +4.4007
Biggest up move:
4.96 / .94 = 5.27 st. dev.
1 time in 7,000,000
Biggest down move:
6.66 / .94 = 7.08 st. dev.
1 time in 600,000,000,000
(1 in 600 billion)
number of occurrences
0
50
100
150
200
250
300
350
400
450
500
-7%
-6%
-5%
-4%
-3%
-2%
-1%
0%
1%
2%
3%
4%
5%
6%
daily price change (nearest ¼ percent)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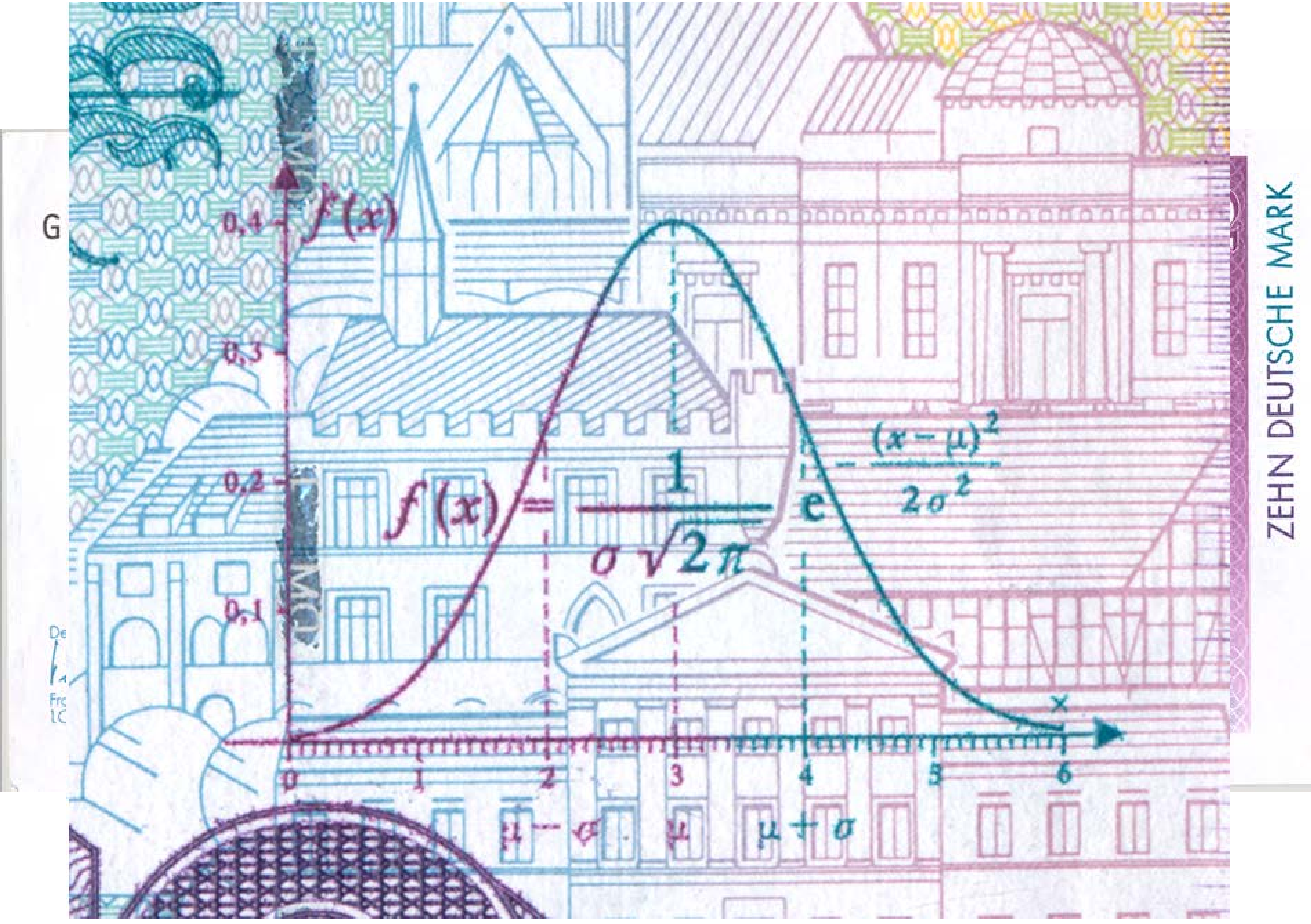
f(x)
0,4
0,3
0,2
0,1
$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
x
0
1
2
3
4
5
6
$\mu - \sigma$
μ
$\mu + \sigma$
ZEHN DEUTSCHE MARK