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Understanding risk

- Market risk, in its many forms, is an expression of event probabilities
- Probability is unobservable, hence a matter of opinion
- Markets react to perceived risk and, as we know, option prices are highly sensitive to this
- Event probabilities inferred from market prices is what is meant by market implied risk (a.k.a. Risk Neutral Measure)
Interplay between risk and market prices

- A feedback loop exists between risk and market prices
- Let’s explore some formalities of constructing the Risk Neutral Measure (RNM) and how to ensure it’s done right
Example: market probability of Fed rate policy action

- Linear relationship between Fed Fund futures price and FOMC policy action
- One price, one unknown with model independent (mostly) relationship
- Hard to mess up
- Easily determined market consensus of the probability of Fed Rate policy action
Equity/options market implied event probabilities

Wouldn’t it be nice?

- To have a precise answer to questions such as “what’s the market probability SPX drops $x\%$ next month?”
  - Why don’t we see much of this?
- Because unlike FOMC actions and the VIX calculation, options quotes do not offer a straightforward way to infer a consensus view
  - How come?
Equity options landscape

- Unlike FOMC actions, equity events are not simply binary
- Potentially infinitely many SPX prices can be realized
- Multiple horizons (1d, 1w, 2w, ..., 1m, 2m, ..., 1y, 2y, ...) instead of simply a single timeframe
- Much more complicated continuum of probability distributions needed (a.k.a. Volatility Surface)
Volatility surface

- This is a well known concept, hasn’t this been settled already?
  - Not quite
- There are typically hundreds (sometimes thousands, e.g. SPX) of market option quotes
  - Yet this is still finite and therefore a massively underdetermined system, considering there are infinite unknown events across a continuum of horizons
  - Hence a model is required
- *Volatility surface consensus hinges on model consensus*
Example: Heston model

- Widely accepted equity option model
- Must calibrate model to quotes, similar to determining probability of FOMC action from Fed Fund futures price
- Heston has 5 parameters to calibrate to hundreds (or thousands) of market quotes (overdetermined)
- How can this be achieved?
  - Strictly speaking, it is **not** possible
  - It’s a matter of choosing how/where to compromise fit
Heston model

Calibration alternatives

- **Heston 1**
  - Fit to 8 quotes in strike range 2475-3625
  - Volatility weighted RMSE 0.84%
  - Misprices 231 of 443 quotes in strike range 1000-3900

- **Heston 2**
  - Fit to 8 quotes in strike range 2475-3625
  - Premium weighted RMSE 0.27
  - Misprices 97 of 443 quotes in strike range 1000-3900

- **Heston 3**
  - Fit to 13 quotes in strike range 1700-3625
  - Volatility weighted RMSE 2.32%
  - Misprices 258 of 443 quotes in strike range 1000-3900
Heston model calibrations – distributions and statistics

### Probability Distributions

<table>
<thead>
<tr>
<th>Strike</th>
<th>Heston 1</th>
<th>Heston 2</th>
<th>Heston 3</th>
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<tbody>
<tr>
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### Summary Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Heston 1</th>
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<th>Heston 3</th>
</tr>
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<tbody>
<tr>
<td>VIX-like¹ (16.16% actual)</td>
<td>15.86%</td>
<td>15.81%</td>
<td>14.83%</td>
</tr>
<tr>
<td>95% VaR</td>
<td>-8.75%</td>
<td>-8.90%</td>
<td>-7.33%</td>
</tr>
<tr>
<td>99% VaR</td>
<td>-17.16%</td>
<td>-16.01%</td>
<td>-17.98%</td>
</tr>
<tr>
<td>Probability SPX &lt; -20%</td>
<td>0.58%</td>
<td>0.39%</td>
<td>0.76%</td>
</tr>
<tr>
<td>Probability -20% &lt; SPX &lt; -10%</td>
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<tr>
<td>Probability -2% &lt; SPX &lt; 0%</td>
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<tr>
<td>Probability 0% &lt; SPX &lt; +2%</td>
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¹The official VIX calculation interpolates a 30 day time horizon from the two surrounding expirations. The numbers in the table skip this step and instead provide the 31 day time horizon found from the single 3/6/2020 expiration. Officially published VIX on 2/4/2020 was 16.05%.
Even if participants agree on a single model such as Heston, consensus is impossible due to calibration choice.

No established models are capable of fitting option prices anyhow.

How can risk statistics be trusted when each has a variety of model/calibration values?
What about VIX?

- Does this mean VIX is a nonsense number?
- **No**, it is based on the Carr-Madan formula for variance swap pricing and is (mostly) model independent, permitting a consensus value
- Single number offers limited information about full risk spectrum
- Has no answer to questions like, “What is the probability SPX will drop $x\%$?”
Establishing trustworthy market event probabilities

What will it take?

- Accurate and reliable forward pricing
- Outlier identification
- Consistent with remaining option quotes
- Trouble is, with infinitely many ways to fit available quotes, there is no right answer
- Unless a unique “best” probability distribution can be identified within this set
- Field of mathematics called calculus of variations does precisely that, as long as there is a sensible metric ranking any two candidates
- Used in wide range of applications such as engineering, flight paths, physics
- Fairly common exercise in fixed income modeling as well (infinite dimensional underlying and all)
Constructing a probability distribution ranking metric

Both fit available option quotes, which one is better?
Ideal fit – calibration results

- Fit to all quotes in strike range 100-4800
- Misprices 0 of 443 quotes
Ideal fit – distribution and statistics

Probability Distribution

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Potential applications

- Improve pricing accuracy and market consistency
  - Less liquid expiration dates
  - Far out-of-the-money options
  - FLEX options
  - Exotic payoff profiles
  - Outlier detection
- Clean volatility surface as input to other processes
- High granularity risk estimation
  - Price range probability
  - Value-at-Risk
  - Expected shortfall/tail loss
  - Forecasting
Thank you

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