Beyond VIX: Data-Driven Market Implied Risk

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Milliman

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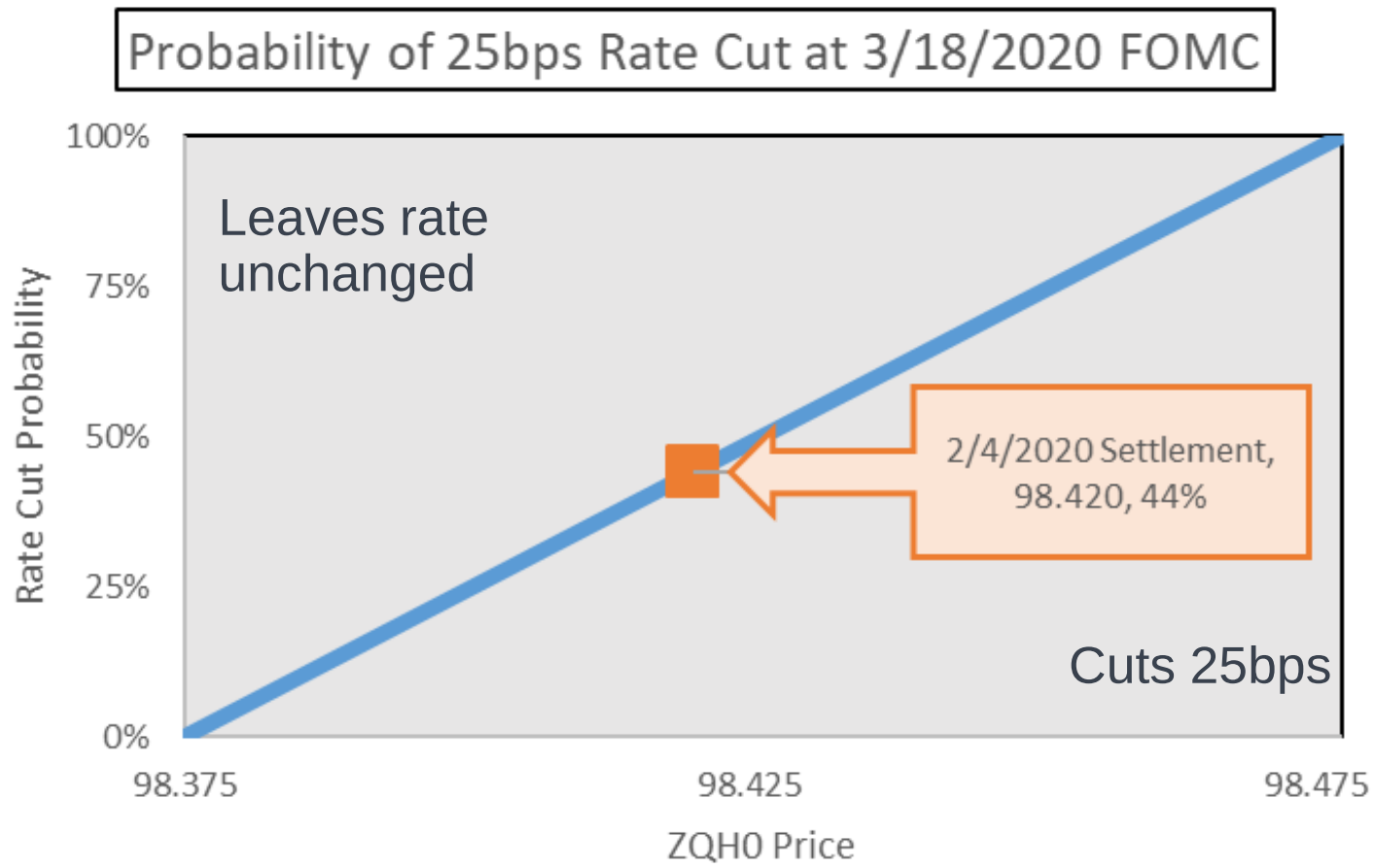
Understanding risk

- Market risk, in its many forms, is an expression of event probabilities
- Probability is unobservable, hence a matter of opinion
- Markets react to perceived risk and, as we know, option prices are highly sensitive to this
- Event probabilities inferred from market prices is what is meant by market implied risk (a.k.a. Risk Neutral Measure)

Interplay between risk and market prices

- A feedback loop exists between risk and market prices
- Let's explore some formalities of constructing the Risk Neutral Measure (RNM) and how to ensure it's done right

Example: market probability of Fed rate policy action



- Linear relationship between Fed Fund futures price and FOMC policy action
- One price, one unknown with model independent (mostly) relationship
- Hard to mess up
- Easily determined market consensus of the probability of Fed Rate policy action

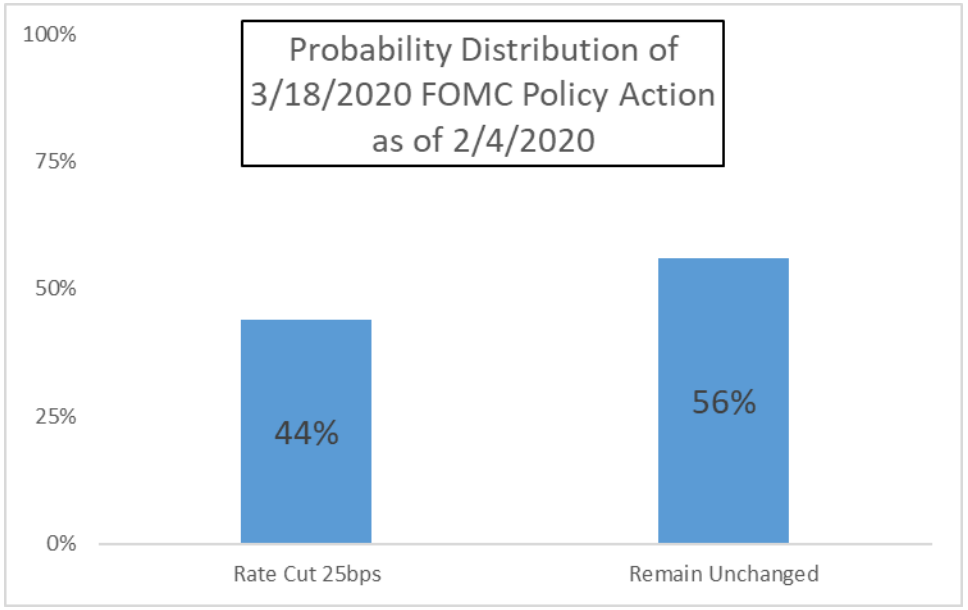
Equity/options market implied event probabilities

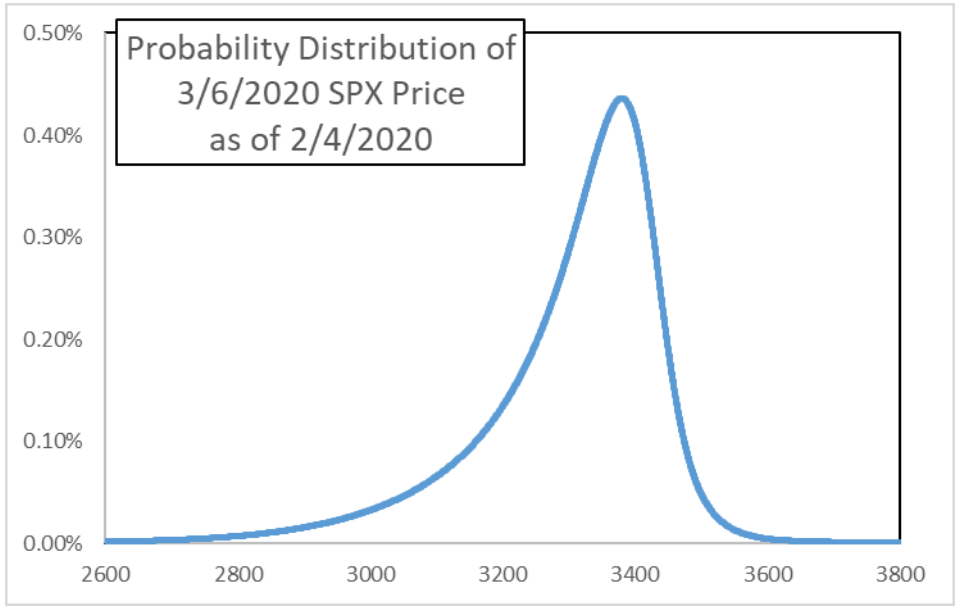
Wouldn't it be nice?

- To have a precise answer to questions such as "what's the market probability SPX drops x% next month?"
  - Why don't we see much of this?
- Because unlike FOMC actions and the VIX calculation, options quotes do not offer a straightforward way to infer a consensus view
  - How come?

Equity options landscape

- Unlike FOMC actions, equity events are not simply binary
- Potentially infinitely many SPX prices can be realized
- Multiple horizons (1d, 1w, 2w, …, 1m, 2m, …, 1y, 2y, …) instead of simply a single timeframe
- Much more complicated continuum of probability distributions needed (a.k.a. Volatility Surface)





Volatility surface

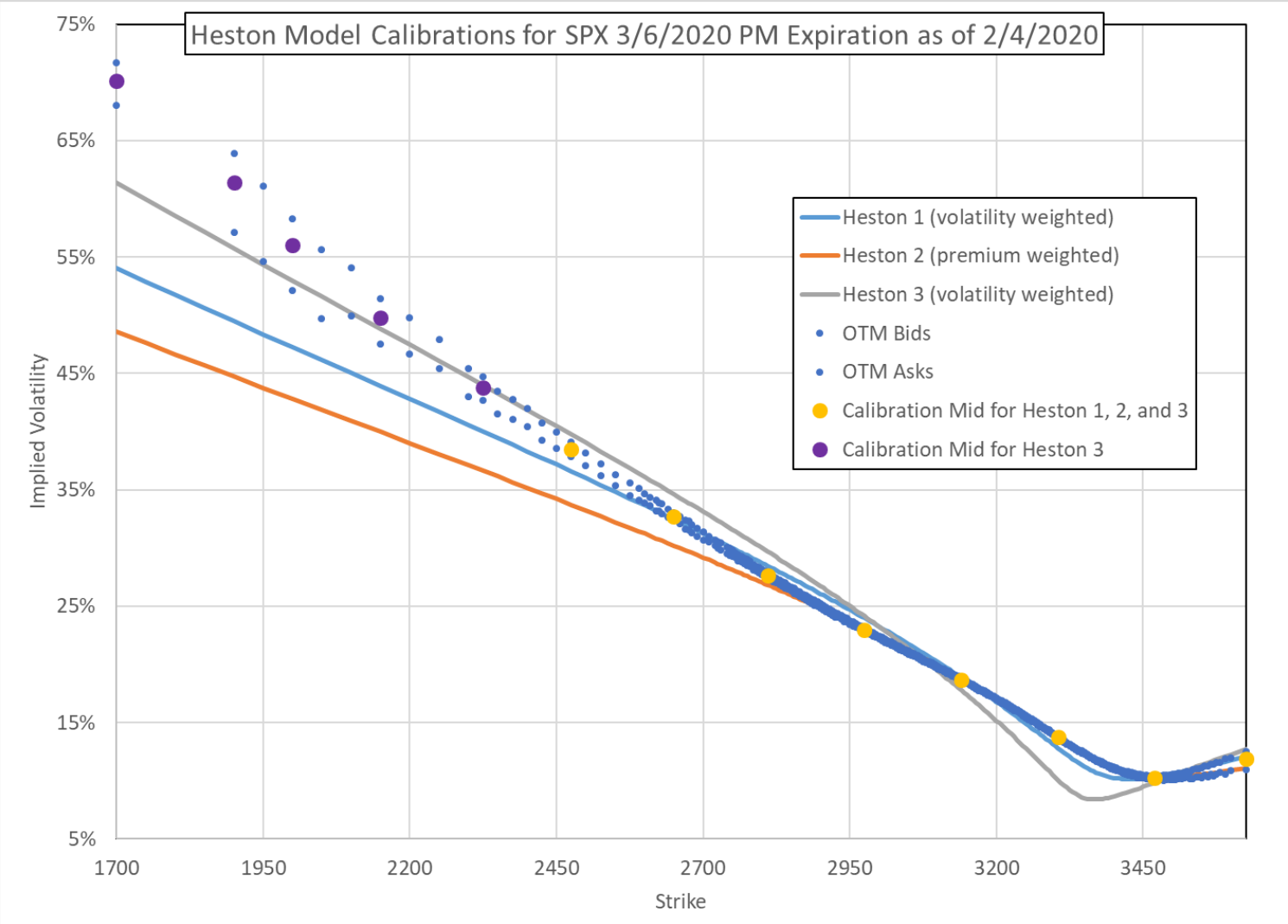
- This is a well known concept, hasn't this been settled already?
  - Not quite
- There are typically hundreds (sometimes thousands, e.g. SPX) of market option quotes
  - Yet this is still finite and therefore a massively underdetermined system, considering there are infinite unknown events across a continuum of horizons
  - Hence a model is required
  - ***Volatility surface consensus hinges on model consensus***

Example: Heston model

- Widely accepted equity option model
- Must calibrate model to quotes, similar to determining probability of FOMC action from Fed Fund futures price
- Heston has 5 parameters to calibrate to hundreds (or thousands) of market quotes (overdetermined)
- How can this be achieved?
  - Strictly speaking, it is **not** possible
  - It's a matter of choosing how/where to compromise fit

Heston model

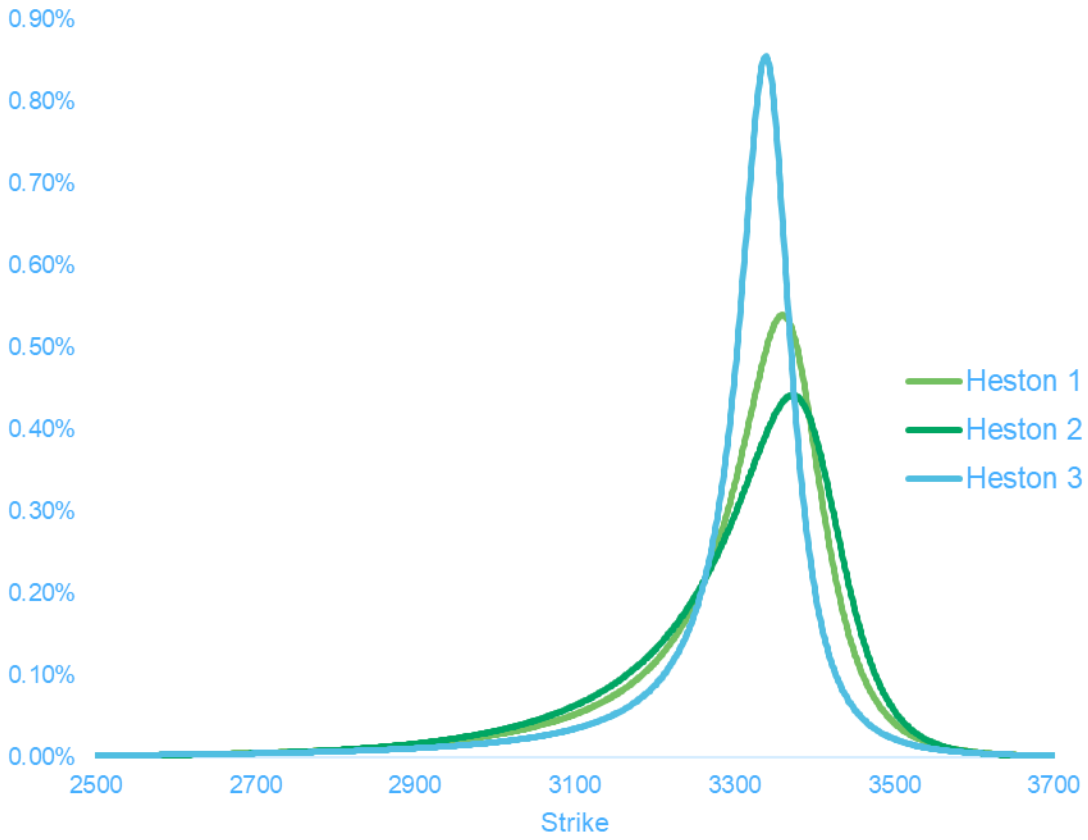
Calibration alternatives



- Heston 1
  - Fit to 8 quotes in strike range 2475-3625
  - Volatility weighted RMSE 0.84%
  - Misprices 231 of 443 quotes in strike range 1000-3900
- Heston 2
  - Fit to 8 quotes in strike range 2475-3625
  - Premium weighted RMSE 0.27
  - Misprices 97 of 443 quotes in strike range 1000-3900
- Heston 3
  - Fit to 13 quotes in strike range 1700-3625
  - Volatility weighted RMSE 2.32%
  - Misprices 258 of 443 quotes in strike range 1000-3900

Heston model calibrations – distributions and statistics

Probability Distributions



Summary Statistics

Statistic	Heston 1	Heston 2	Heston 3
VIX-like[1] (16.16% actual)	15.86%	15.81%	14.83%
95% VaR	-8.75%	-8.90%	-7.33%
99% VaR	-17.16%	-16.01%	-17.98%
Probability SPX < -20%	0.58%	0.39%	0.76%
Probability -20% < SPX < -10%	3.34%	3.51%	2.45%
Probability -10% < SPX < -5%	6.81%	8.14%	4.50%
Probability -5% < SPX < -2%	10.88%	12.31%	8.33%
Probability -2% < SPX < 0%	16.40%	15.71%	18.88%
Probability 0% < SPX < +2%	31.41%	25.51%	47.03%
Probability +2% < SPX < +5%	26.86%	29.81%	15.90%
Probability SPX > +5%	3.73%	4.61%	2.15%

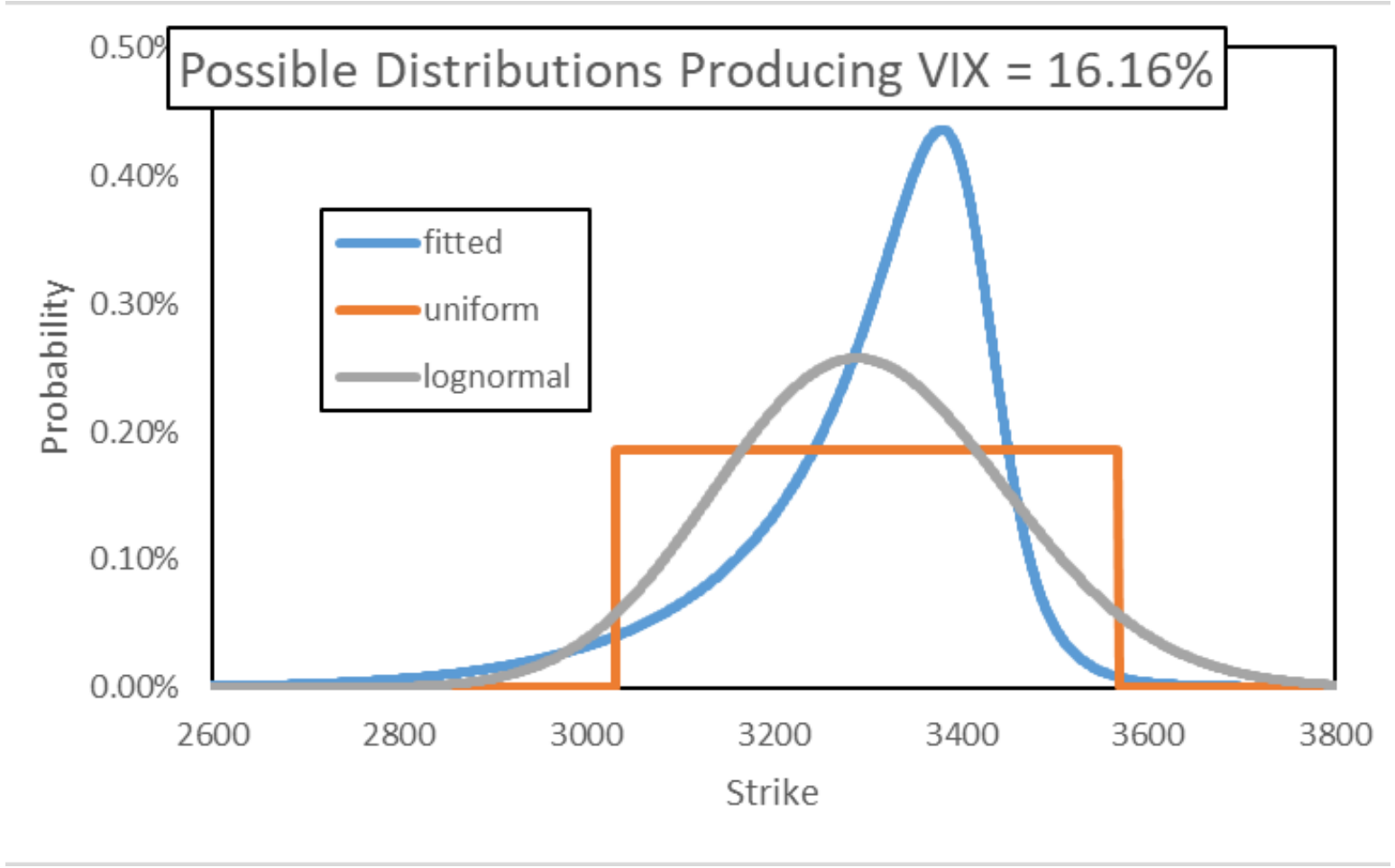
[1]The official VIX calculation interpolates a 30 day time horizon from the two surrounding expirations. The numbers in the table skip this step and instead provide the 31 day time horizon found from the single 3/6/2020 expiration. Officially published VIX on 2/4/2020 was 16.05%.

Interpreting model results

- Even if participants agree on a single model such as Heston, consensus is impossible due to calibration choice
- No established models are capable of fitting option prices anyhow
- How can risk statistics be trusted when each has a variety of model/calibration values?

What about VIX?

- Does this mean VIX is a nonsense number?
- **No,** it is based on the Carr-Madan formula for variance swap pricing and is (mostly) model independent, permitting a consensus value
- Single number offers limited information about full risk spectrum
- Has no answer to questions like, “What is the probability SPX will drop x%?”



Establishing trustworthy market event probabilities

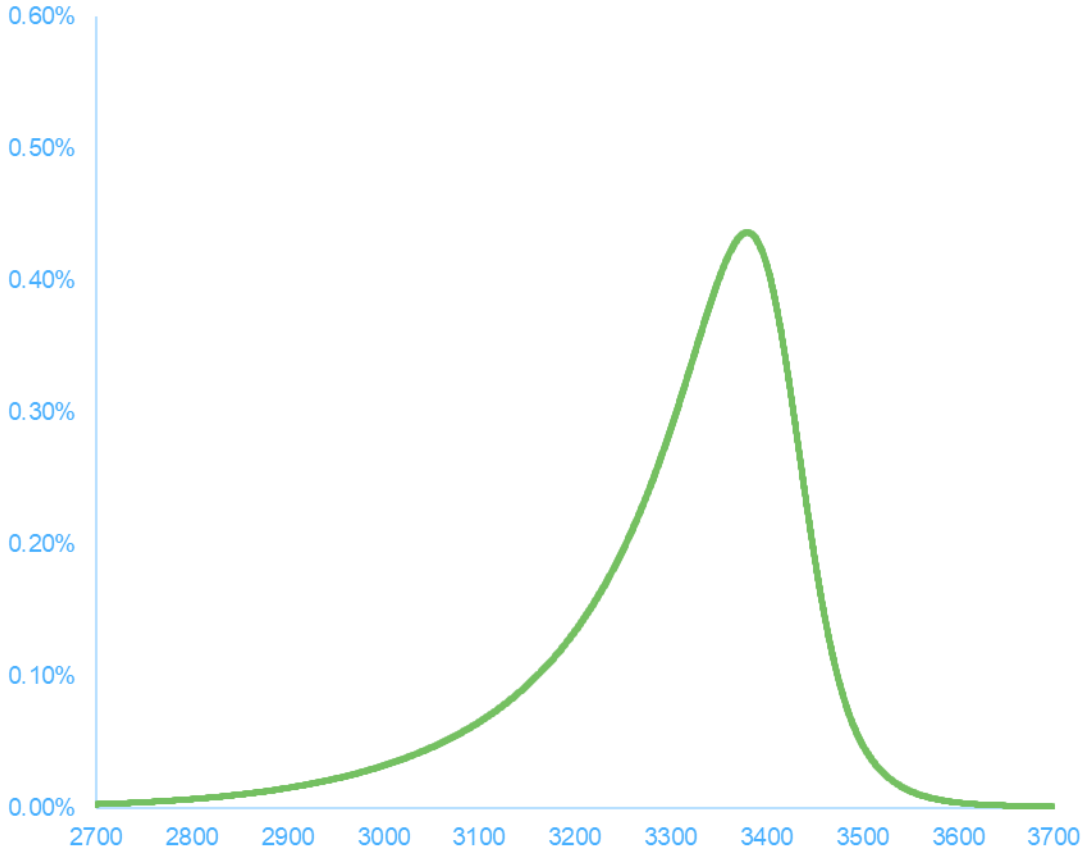
What will it take?

- Accurate and reliable forward pricing
- Outlier identification
- Consistent with remaining option quotes
- Trouble is, with infinitely many ways to fit available quotes, there is no right answer
- Unless a unique “best” probability distribution can be identified within this set
- Field of mathematics called *calculus of variations* does precisely that, as long as there is a sensible metric ranking any two candidates
- Used in wide range of applications such as engineering, flight paths, physics
- Fairly common exercise in fixed income modeling as well (infinite dimensional underlying and all)

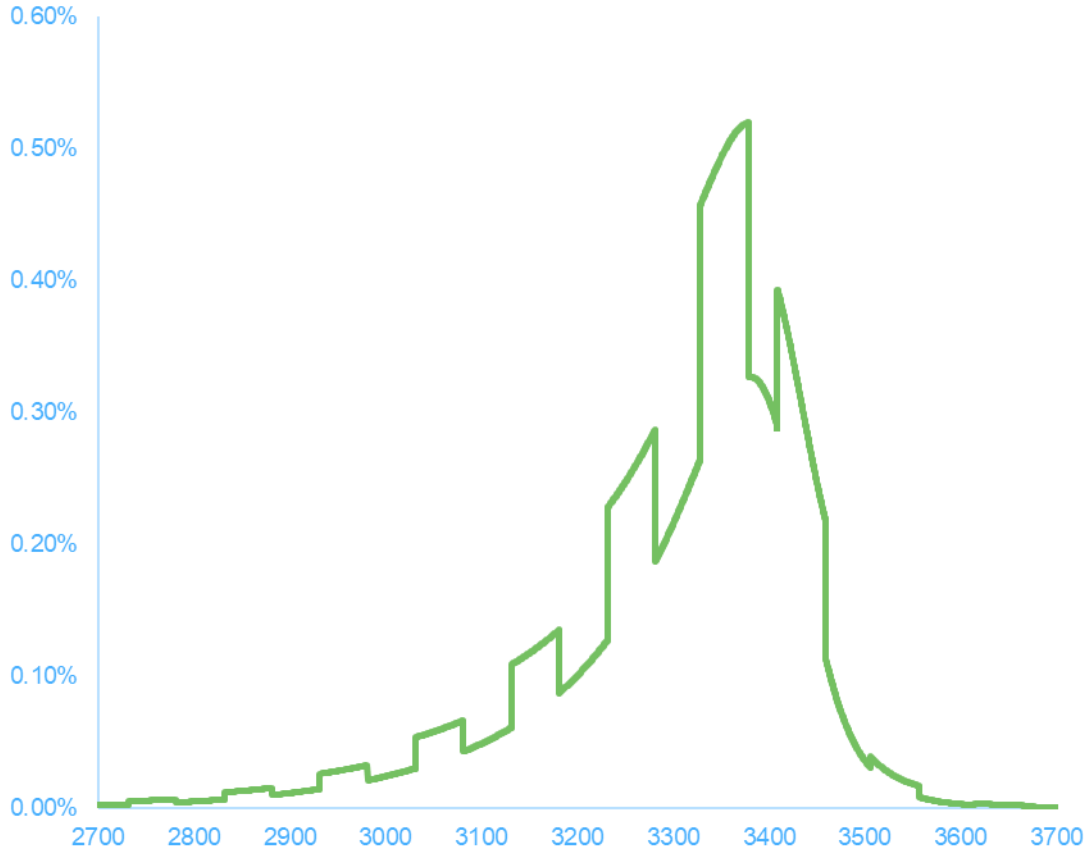
Constructing a probability distribution ranking metric

Both fit available option quotes, which one is better?

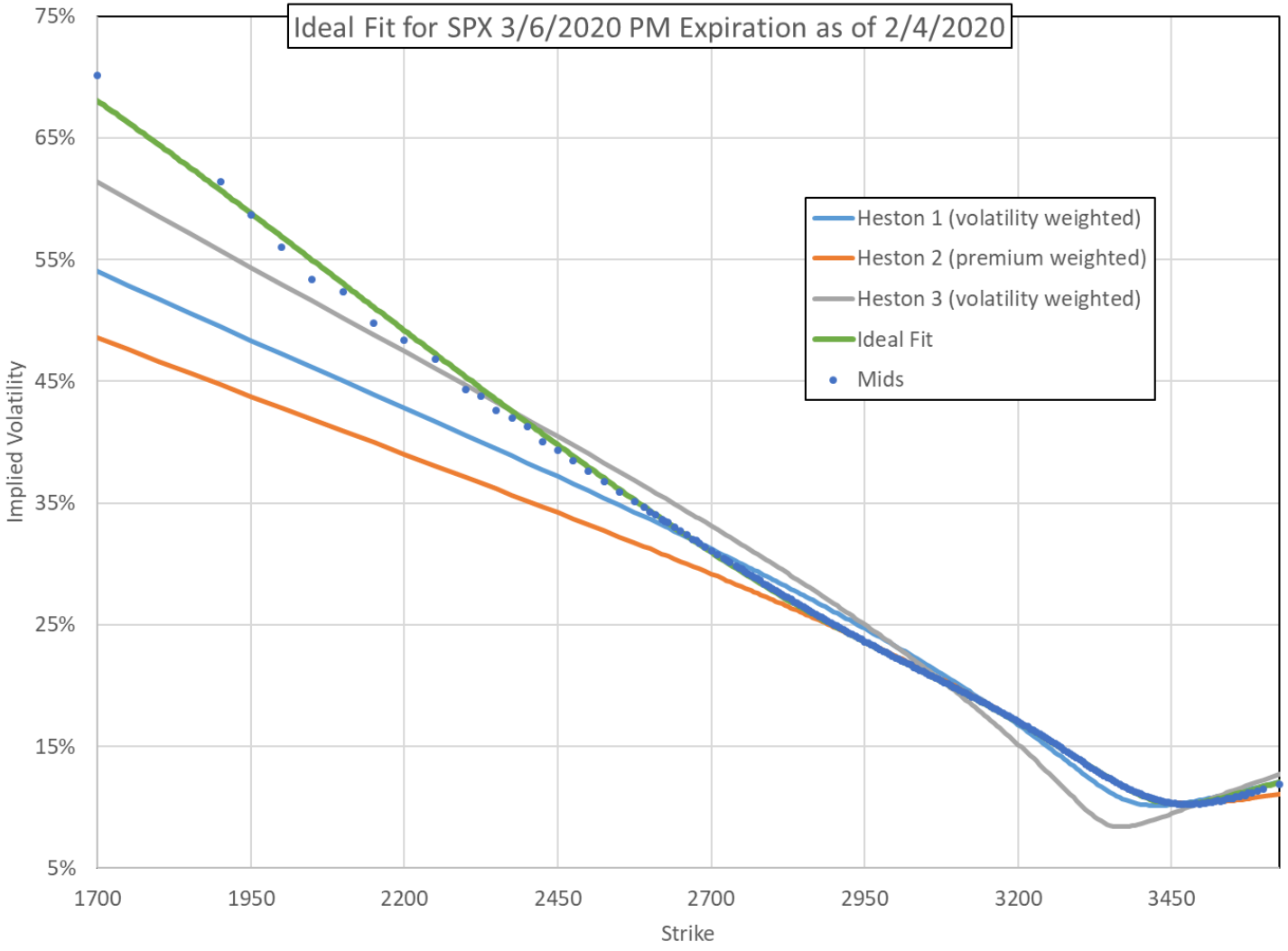
Candidate A



Candidate B



Ideal fit – calibration results

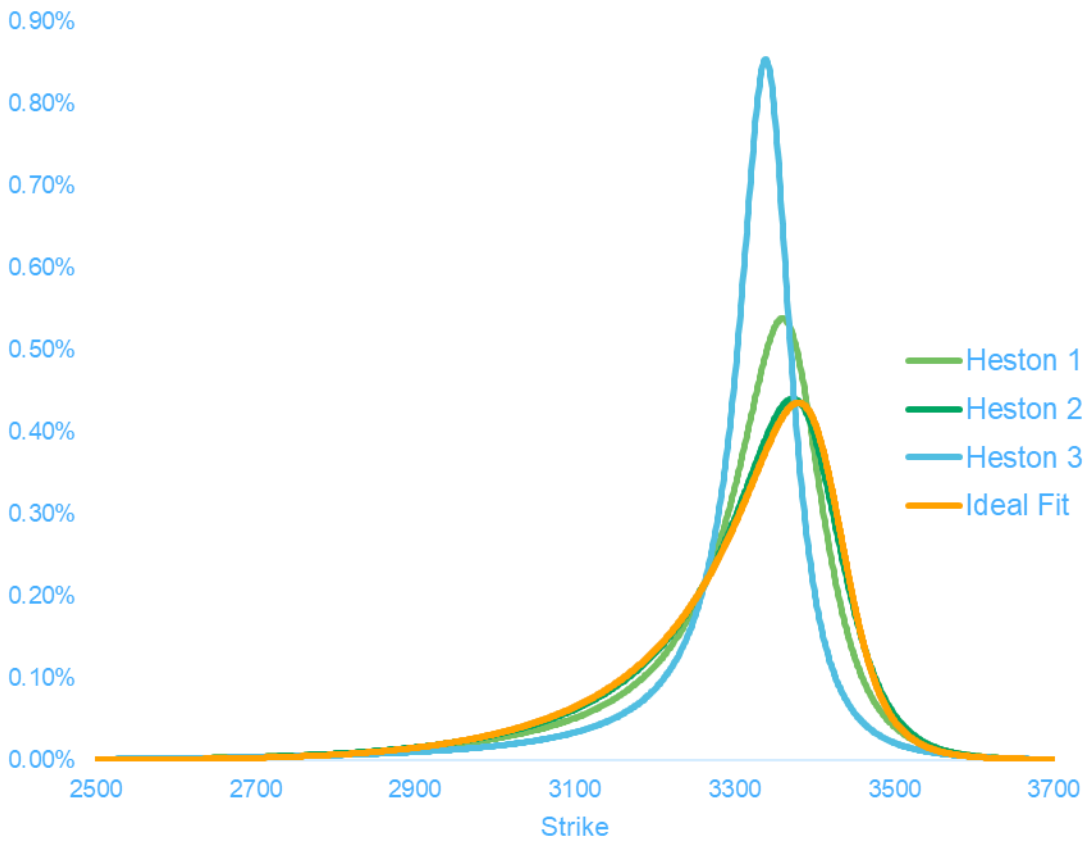


Ideal fit

- Fit to all quotes in strike range 100-4800
- Misprices 0 of 443 quotes

Ideal fit – distribution and statistics

Probability Distribution



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Potential applications

- Improve pricing accuracy and market consistency
  - Less liquid expiration dates
  - Far out-of-the-money options
  - FLEX options
  - Exotic payoff profiles
  - Outlier detection
- Clean volatility surface as input to other processes
- High granularity risk estimation
  - Price range probability
  - Value-at-Risk
  - Expected shortfall/tail loss
  - Forecasting

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Thank you

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